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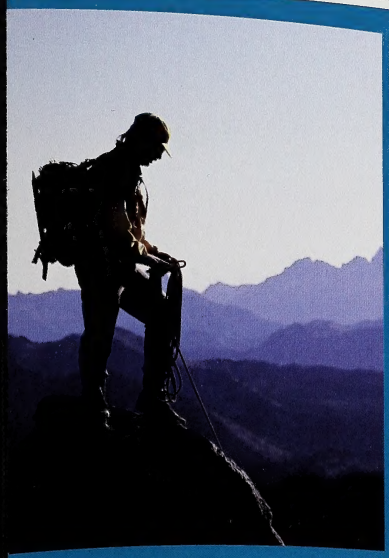
MATHEMATICS 14



Module 2
PATTERNS and EQUATIONS

Learning Technologies Branch

Alberta LEARNING

MATHEMATICS 14

Module 2

PATTERNS and EQUATIONS

Mathematics 14
Module 2: Patterns and Equations
Student Module Booklet
Learning Technologies Branch
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The Learning Technologies Branch acknowledges with appreciation the Alberta Distance Learning Centre and Pembina Hills Regional Division No. 7 for their review of this Student Module Booklet.

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome to MATHEMATICS 14

Mathematics 14 contains five modules. You should work through the modules in order (from 1 to 5) because concepts and skills introduced in one module will be reinforced, extended, and applied in later modules.

Module 1
NUMBER

Module 2
PATTERNS and EQUATIONS

Module 3
FRACTIONS, RATIO, and PERCENT

Module 4
MEASUREMENT

Module 5
GEOMETRY

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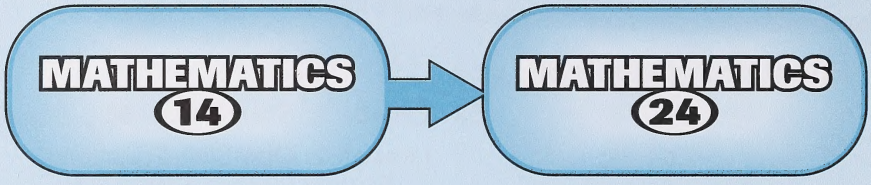
COURSE FEATURES

The Mathematics 14–24 Program

Mathematics 14 is the first course in the Mathematics 14–24 sequence of courses. If you successfully complete each of these five-credit courses, you will meet the minimum requirements in mathematics for an Alberta high school diploma.



The Mathematics 14–24 sequence is designed for students whose needs, interests, and abilities focus on basic mathematical understanding. This course sequence emphasizes the acquisition of practical life skills and proficiency in using mathematics to solve problems, adapt to change, interpret information, and build on previous knowledge.

Consult your teacher or counsellor for the latest information. Also, if you have access to the Internet, you can find out more about Mathematics 14 and high school requirements at the Alberta Learning website.

http://www.learning.gov.ab.ca

Take the time to look through the Student Module Booklets and the Assignment Booklets and notice the following design features:

- Each module has a Module Overview, Module Summary, and Review.
- Each module has several sections. Each section focuses on a big idea that is central to the topic being learned in the module.
- Each section has several lessons.
- Each module has a Glossary and an Answer Key in the Appendix. In several modules there are also special pull-out pages in the Appendix.
- Each module references the CD that accompanies your *Continuum* textbook.

Required Resources

There are no spaces provided in the Student Module Booklets for your answers. This means you will need a binder and loose-leaf paper or a notebook to do your work.

In order to complete the course, you will need a copy of the Mathematics 14 textbook, *Continuum*, a scientific calculator (such as the Texas Instruments TI-30X IIS), and various manipulatives (pattern blocks and fraction blocks). For your convenience, cut-out fraction blocks are provided in the Appendix of Module 3.

Pattern blocks and fraction blocks are available from the Learning Resources Centre. As of 2003, the product codes for these items were 161901 and 408288, respectively. Check for the latest ordering information at the LRC website.

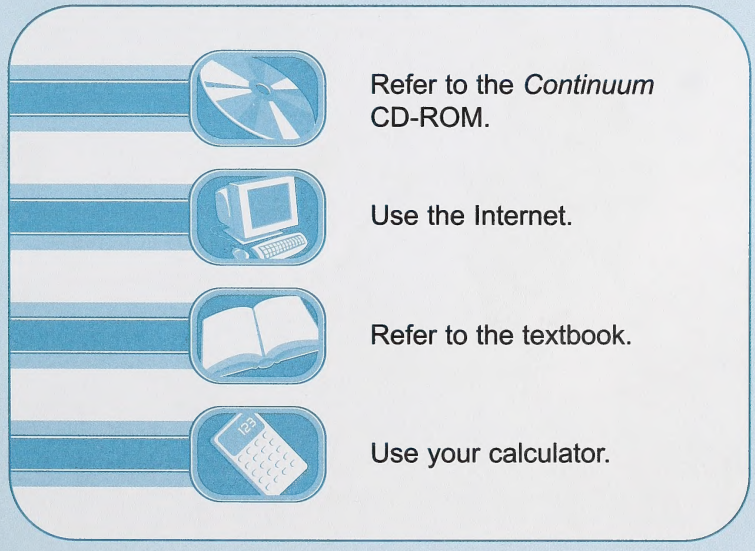
http://www.lrc.learning.gov.ab.ca

If you wish to complete the optional computer activities, you must have access to a computer that is connected to the Internet.

You will also need access to a computer to view material on the CD-ROM that accompanies your *Continuum* textbook.

Visual Cues

For your convenience, the most important mathematical rules and definitions are highlighted. Icons are also used as visual cues. Each icon tells you to do something.



ASSESSMENT AND FEEDBACK

The Mathematics 14 course is carefully designed to give you many opportunities to discover how well you are doing. In every lesson you will be asked to turn to the Appendix to check your answers. Completing the lessons and comparing your answers to the suggested answers in the Appendix will help you better understand math concepts, develop math skills, and improve your ability to communicate mathematically and solve problems.

If you are having difficulty with a lesson, refer to the Answer Key in the Appendix for hints or help. As well as giving suggested answers to the questions, the Answer Key gives you more information about the questions.

Twice in each module you will be asked to give your teacher your completed assignments to mark. Your teacher will give you feedback on how you are doing.

After your teacher marks an assignment, be sure to review your teacher's comments and correct any errors you made.

There will be a Final Test at the end of the course. You can prepare for the Final Test by completing the Review at the end of each module.

MODULE OVERVIEW

The artistic arrangement of the bricks is one of the striking features of the walkway shown above. Interesting walkways can be designed even with only a single colour or style of brick. Your eyes are drawn to the slightly uneven surface. As you study the brickwork, you see the patterns the bricklayers used. These patterns contain repeated elements. When the bricklayers were planning the walkway, they had to calculate how many bricks of each shape and colour they were going to use to complete the design. This would be an interesting activity to participate in. There is a whole lot of practical mathematics involved.

In this module you will be looking for and analysing patterns. Sometimes you will use equations to help you. You will also solve equations that come up in everyday situations.

Module 2
PATTERNS and EQUATIONS

Section 1
PATTERNS and PUZZLES

Section 2
EQUATIONS and GRAPHS

Your mark on this module will be determined by how well you complete the two Assignment Booklets.

The mark distribution is as follows:

Assignment Booklet 2A

Section 1 Assignment	30 marks

Assignment Booklet 2B

Section 2 Assignment	30 marks
Final Module Assignment	40 marks
Total	100 marks

When doing the assignments, work slowly and carefully. Be sure you attempt each part of the assignments. If you are having difficulty, you may use your course materials to help you, but you must do the assignments by yourself.

You will submit Assignment Booklet 2A to your teacher before you begin Section 2. You will submit Assignment Booklet 2B to your teacher at the end of this module.

SECTION 1

Patterns and Puzzles

At highland games, the kilts are as distinctive as the events. It seems like there is no limit to the number of tartan patterns used in the kilts. Some of the differences among these patterns are quite subtle, other differences are more obvious.

Tartan patterns have developed over the centuries. In the beginning, different regions of Scotland had different tartans. Over the years, each clan or family made claim to its own pattern. There are also specific tartans for dress, mourning, sports and outdoor activities, and military personnel.

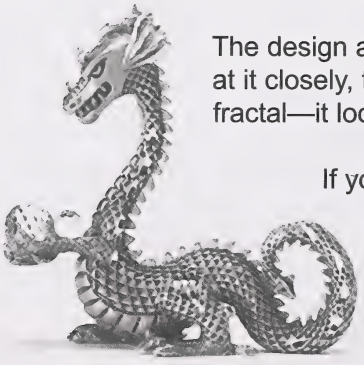
A quick check on the Internet will show you that there are now nearly 3000 different registered tartan patterns. For more information about tartans, do an Internet search using the keywords *history of tartan*.

In this section you will be studying diagrams, lists, and problem situtations to find patterns. Often, the pattern may not be immediately obvious.

LESSON 1

Recognizing Patterns

Today you will study how to recognize and describe patterns.

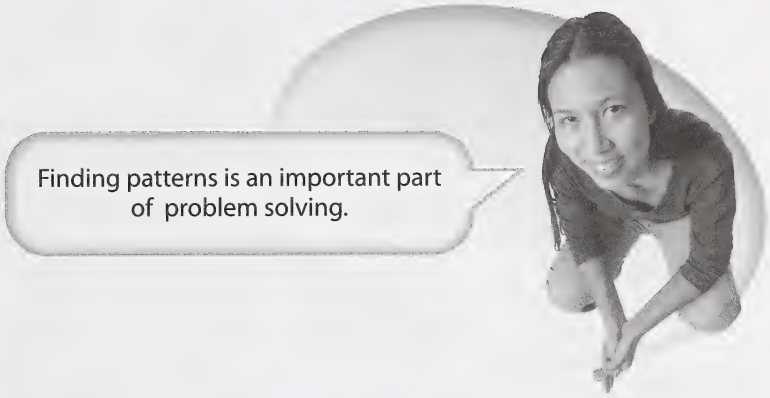


The design above is called a dragon fractal. When you look at it closely, the blue part reveals why it is called a dragon fractal—it looks a lot like a dragon standing on its rear legs.

If you study the **pattern**, you will see many smaller parts that look quite similar to the whole design. That is why it is called a fractal. Fractals are objects that have this self-similarity property.

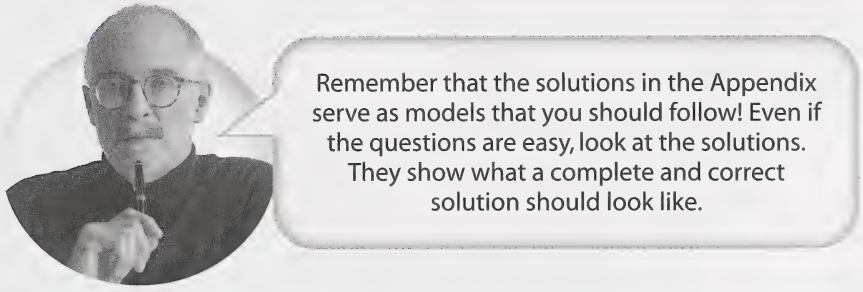
Patterns are an important part of mathematics. Some are subtle and are quite hard to find. Others are bold and easy to see. This lesson will help you develop your pattern-finding skills.

Turn to page 39 in your textbook, and read the introduction to "Tutorial 1: Recognizing Patterns." Notice the three-step process you will be exploring in this lesson (recognize, extend, and describe).



1. Turn to pages 39 and 40 in your textbook. Complete "Investigation" questions a. through e. This investigation is an introduction to determining patterns and expressing them in algebraic form.

Check your answers on pages 58 and 59 in the Appendix.



The hints in "Investigation" were designed to help you find the equation to describe the pattern. Then you used this equation to make predictions beyond the numbers in the table. Without the equation, you would have to extend the table term by term to arrive at the same result. Extending a table can take a lot of time. Using an equation is often quicker and much more efficient.

Example

Ralph works in a restaurant that serves fried chicken. To explain why it costs extra to have additional drumsticks, he used a table. To get the extra drumsticks, you need more chickens, and someone has to use the rest of the extra chicken. If you want something special, you have to pay extra. What pattern can be seen in Ralph's table? Write an equation to represent the pattern.

Number of Chickens	Number of Drumsticks
1	2
2	4
3	6

Let c represent the number of chickens. Let d represent the number of drumsticks. For each extra chicken, there are two additional drumsticks. The number of drumsticks is twice the number of chickens. In algebraic form, this is written as $d = 2 \times c$ or $d = 2c$.

Example

Alli was watching an ant at a picnic. It covered the distance from the edge of the blanket to the potato salad in about 20 s. Alli determined the ant was travelling about 5 cm each second.

The algebraic form of this pattern is $d = 5t$. How would you use this equation to figure out how far the ant could travel over different time periods? Record the ant's distances in the following table.

Time (s)	Distance (cm)
1	
3	
6	
10	
25	

To find these distances, the procedure you use is called **substitution** for the variable t.

Time (s)	Distance (cm) $(d = 5t)$
1	$5 \times 1 = 5$
3	$5 \times 3 = 15$
6	$5 \times 6 = 30$
10	$5 \times 10 = 50$
25	$5 \times 25 = 125$

Now it's your turn to find and describe patterns.

2. Turn to pages 41 to 44 in your textbook. Solve questions 1 to 5 of "Put into Practice." You will find the margin notes and diagrams useful in solving these questions.

Check your answers on pages 59 to 65 in the Appendix.

Turn to the Section 1 Assignment in Assignment Booklet 2A. Answer question 1.

LESSON 2

Graphing Patterns

Today you will study how to visualize and graph patterns.

One natural pattern that you probably aren't familiar with is the pattern of ocean tides. Tides and their patterns are very important to seafaring people. The ancient Phoenicians' trade routes 3000 years ago included Britain. This was a sea voyage from what is now Lebanon to Cornwall, in southwest Britain. To travel from port to port, it is necessary to be able to predict the tides. The tracking of tides has an extremely long and important history.

Being able to predict when the tide will be low or high can be a matter of life and death. Harbours may be too shallow for large ships at low tide. Rocks can be too near the surface, so boats may be damaged. Being caught on a beach as the tide comes in can be deadly.

Recording observations is important in lots of other situations. How the observations are recorded and used varies with the situation. Observations are often kept as lists of numbers or graphs. You will look at how to make graphs from lists of numbers in this lesson; but, first, you will review plotting points on the co-ordinate plane.

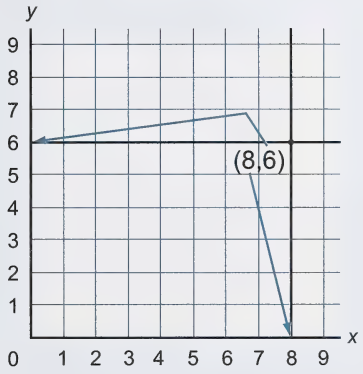
Looking Back

Turn to page 45 in your textbook. Read the explanation of graphing points on the co-ordinate plane.

Example

Louis missed the math class about graphing points on the co-ordinate plane. Yaggi is trying to help him graph the point $(8, 6)$. What should Yaggi say to make sure Louis understands?

Yaggi wants Louis to look at the two numbers separately. The first number, 8, indicates how far to move along the *x*-axis. The second number, 6, indicates how far to move along the *y*-axis.



The point is $(8, 6)$. To reach this point, you move 8 units to the right along the *x*-axis. Then, move 6 units up along the *y*-axis. The point is located where the highlighted lines meet.

1. On page 45 in your textbook, complete questions a. and b. of "Put into Practice. "

2. Turn to page 46 in your textbook. Work through "Investigation." You will be graphing a **linear relationship**.

Check your answer on pages 66 and 67 in the Appendix.

Example

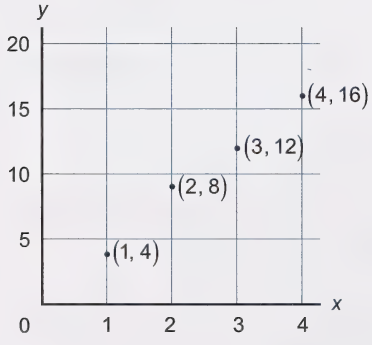
Joey and his little sister, Emily, are waiting for their dad outside the grocery store. While they are waiting, Emily starts to count the number of tires in the parking lot. Joey writes down what she counts. The results are shown in this table. What ordered pairs does this table suggest?

Number of Cars	Number of Tires
1	4
2	8
3	12
4	16

The ordered pairs are $(1, 4), (2, 8), (3, 12),$ and $(4, 16)$.

Number of Cars	Number of Tires	
1	4	$(1, 4)$
2	8	$(2, 8)$
3	12	$(3, 12)$
4	16	$(4, 16)$

Joey thinks that graphing these points would be good practice. He only has a small piece of graph paper, so, to fit all the data, he uses a different scale for the *y*-axis than for the *x*-axis.



4 units along *x* and 16 units along *y*

3 units along *x* and 12 units along *y*

2 units along *x* and 8 units along *y*

1 unit along *x* and 4 units along *y*

Now you will get a chance to find ordered pairs and graph them.

3. Turn to pages 47 and 48 in your textbook. Answers questions 1, 2, and 3 of "Put into Practice."

Check your answers on pages 67 to 69 in the Appendix.

Turn to the Section 1 Assignment in Assignment Booklet 2A. Answer question 2.

LESSON 3

Points or Lines?

Today you will explore whether graphs should be drawn with points or lines.

When you are watching TV, the image may look crisp and clear, but it's all an illusion! That "solid" picture is really a series of glowing dots. It's not always easy to tell if something is solid or made up of separate elements. In mathematics, as well, graphs are made solid sometimes and with dots at other times. In this lesson you will be learning when to use dots and when to use solid lines.

Turn to page 49 in your textbook. Read the introduction to "Tutorial 4: Points or Line?" Notice how the choice of drawing a point graph or a line graph is made.

1. Turn to pages 49 and 50 in your textbook. Complete the questions in "Investigation."

Check your answers on page 69 in the Appendix.

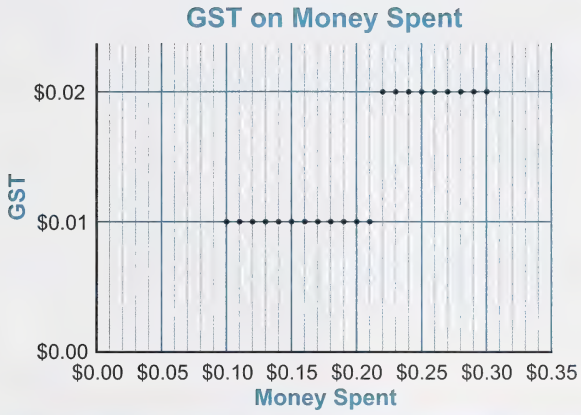
Turn to page 51 in your textbook. Study Example 1 carefully. (**Note:** The values in this example are not realistic. Melon juice is not 3 times as heavy as water.)

Example

Sean has built the following table to show the amount of GST charged on various purchases. He knows the GST is 7% of the amount of a purchase. Draw a graph of the relationship shown in the table.

Amount Spent	Amount of GST	Amount Spent	Amount of GST
$0.10	$0.01	$0.21	$0.01
$0.11	$0.01	$0.22	$0.02
$0.12	$0.01	$0.23	$0.02
$0.13	$0.01	$0.24	$0.02
$0.14	$0.01	$0.25	$0.02
$0.15	$0.01	$0.26	$0.02
$0.16	$0.01	$0.27	$0.02
$0.17	$0.01	$0.28	$0.02
$0.18	$0.01	$0.29	$0.02
$0.19	$0.01	$0.30	$0.02
$0.20	$0.01		

The graph would look like this. It would use points rather than a line because purchases can only be made at penny intervals. Also, notice the step-like shape of the graph. The amount of tax is the same for a number of values, and then it jumps up a cent.



2. Turn to pages 51 and 52 in your textbook. Answer questions 1 to 4 of "Put into Practice." In question 1 you must make a decision about how Carol pays. Does Carol pay to the next higher minute for the time she talks, like most pay-as-you-go cell phone plans; or does she pay for exactly the amount of time she talks? Either is possible, but your answer will depend on your choice.

Check your answers on pages 70 to 72 in the Appendix.

Turn to the Section 1 Assignment in Assignment Booklet 2A. Answer question 3.

LESSON 4

Magic Squares

Today you will learn about magic squares.

Have you ever watched a stage magician? All sorts of seemingly impossible things happen. Solid metal hoops pass through each other. People disappear before your eyes. Rabbits come out of hats. Yet, it is all illusion. There is a simple natural explanation for all of it. In mathematics, there are things that seem to be magic, too. Strange things just seem to happen. In this lesson you will be looking at magic in math.

1. Turn to page 53 in your textbook. Do questions 1 and 2 of "Investigation." You will see what is "magic" about magic squares.

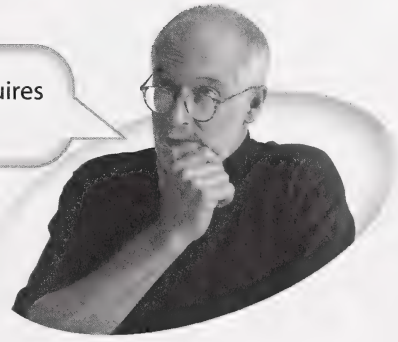
You can find out more about magic squares at the following website:

http://mathworld.wolfram.com/MagicSquare.html

2. Turn to page 54 in your textbook. Complete questions 1 and 2 of "Put into Practice."

Check your answers on pages 72 and 73 in the Appendix.

Turn to page 55 in your textbook. Study "Algebra Connection" carefully.



Example

Complete the following magic square.

11	15	
		9

For a 3-by-3 magic square, the magic sum is 3 times the middle square. The magic sum will be $15 \times 3 = 45$. To solve the square, look for a line of squares where two values are already known.

The coloured square is the one being found.

Square to Find	Calculation	New Partial Magic Square
	The diagonal sum is 45. 45 = 9 + 15 + 45 = 9 + 15 + 21	
	The horizontal sum is 45. 45 = 11 + 15 + 45 = 11 + 15 + 19	
	The vertical sum is 45. 45 = 9 + 19 + 45 = 9 + 19 + 17	
	The horizontal sum is 45. 45 = 21 + 17 + 45 = 21 + 17 + 7	
	The diagonal sum is 45. 45 = 17 + 15 + 45 = 17 + 15 + 13	
	The vertical sum is 45. 45 = 7 + 15 + 45 = 7 + 15 + 23	

Example

You can build a magic square that contains negative numbers. Make a sequence that starts with -7 and then add 4 at each step. Use this sequence to build a magic square.

The sequence is calculated as follows:

$$-7,\ -7+4=-3,\ -3+4=1,\ 1+4=5,\ 5+4=9,\ 9+4=13,$$
$$13+4=17,\ 17+4=21,\ 21+4=25$$

This sequence is $-7,\ -3,\ 1,\ 5,\ 9,\ 13,\ 17,\ 21,\ 25$.

These numbers are placed in the appropriate cells of a 3-by-3 magic square. The cells in the magic square are numbered as shown in the square on the left. -7 goes in cell 1, -3 goes in cell 2, and so on. The result is shown in the square on the right.

8	1	6
3	5	7
4	9	2

21 (8)	-7 (1)	13 (6)
1 (3)	9 (5)	17 (7)
5 (4)	25 (9)	-3 (2)

Each column has a sum of 27.

$$21+1+5=27 \qquad -7+9+25=27 \qquad 13+17+(-3)=27$$

Each row has a sum of 27.

$$21+(-7)+13=27 \qquad 1+9+17=27 \qquad 5+25+(-3)=27$$

Each diagonal has a sum of 27.

$$21+9+(-3)=27 \qquad 5+9+13=27$$

Now it's your turn to work with incomplete magic squares. There are other ways to make squares that are magic—you will explore these, too.

3. Turn to pages 56 to 58 in your textbook. Answer questions 3, 4, 5, 7, 8, 9, 10.a., and 11 of "Put into Practice".

Check your answers on pages 74 to 76 in the Appendix.

Turn to the Section 1 Assignment in Assignment Booklet 2A. Answer question 4.

LESSON 5

Number Tricks and Patterns

Today you will work with number tricks.

ALEX SOKOLOWSKI/WORLD-MYSTERIES.COM

Since the beginning of time, people have experienced patterns all around them. They have seen the seasons repeat. They have seen the full moon wane and return. They have seen day and night reoccur.

Civilizations invented devices to time and predict these events. For short periods of time, they developed hourglasses and sundials. For longer periods of time, they developed calendars. Some calendars were like the one on your wall. Others, like the Mayan calendar shown in the photograph above, were even more accurate.

There are a number of mathematical surprises in our calendar. You are going to explore them, along with other cool tricks, in this lesson.

Turn to page 59 in your textbook. Read the introduction to "Tutorial 2: Number Tricks."

You will be trying several number tricks. Later, you will use algebra to find out why they work.

1. Do questions a. to f. of "Investigation" on page 59 in your textbook.

Check your answers on page 77 in the Appendix.

Turn to page 60 in your textbook. Study the explanation of the trick from "Investigation." The algebra used there shows that no matter what number you pick, the final result is always 1.

Now you will get to work on your own with some number tricks.

2. Turn to page 61 in your textbook. Answer questions 1, 2, and 3 of "Put into Practice." In question 3 you need to know what a **perfect square** is. It is the square of a natural number. For example, $2 \times 2 = 4$, $12 \times 12 = 144$, and $25 \times 25 = 625$ are perfect squares.

Check your answers on pages 77 and 78 in the Appendix.

Turn to page 62 in your textbook. Read the introduction to "Tutorial 3: Calendar Math." It describes one pattern on the calendar page.

You might enjoy this website about calendar math:

http://math.rice.edu/~lanius/Lessons/calen.html

Example

DECEMBER 2003

Sun	Mon	Tues	Wed	Thurs	Fri	Sat
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

Pick a 3-by-3 array from the calendar. Take the middle number and multiply it by 3. Put the result aside for a while. Now pick three numbers from the array. Only one number can come from each row. Only one number can come from each column. What is the sum of these numbers?

Suppose you chose the following square.

11	12	13
18	19	20
25	26	27

Three times 19 is 57.

Possible choices of three numbers and their sums follow. Notice how there is only one number chosen from each row. Also, there is only one number chosen in each column.

11	12	13
18	**19**	20
25	26	**27**

$11+19+27=57$

11	12	13
18	19	**20**
25	**26**	27

$11+20+26=57$

11	**12**	13
18	19	20
25	26	**27**

$12+18+27=57$

11	**12**	13
18	19	**20**
25	26	27

$12+20+25=57$

11	12	13
18	19	20
25	26	27

$13+18+26=57$

11	12	13
18	19	20
25	26	27

$13+19+25=57$

Example

Why do the sums all equal 3 times the middle number?

In algebraic form, the square looks like the following:

N	$N+1$	$N+2$
$N+7$	$(N+7)+1$	$(N+7)+2$
$N+14$	$(N+14)+1$	$(N+14)+2$

Three times the middle number is

$$\begin{aligned} 3\times\left[(N+7)+1\right] &= 3N+21+3 \\ &= 3N+24 \end{aligned}$$

The possible choices of three squares follow.

N	$N+1$	$N+2$
$N+7$	$(N+7)+1$	$(N+7)+2$
$N+14$	$(N+14)+1$	$(N+14)+2$

$$\begin{aligned} N+(N+7)+1+(N+14)+2 &= 3N+7+1+14+2 \\ &= 3N+24 \end{aligned}$$

N	$N+1$	$N+2$
$N+7$	$(N+7)+1$	$(N+7)+2$
$N+14$	$(N+14)+1$	$(N+14)+2$

$$\begin{aligned} N+1+(N+7)+(N+14)+2 &= 3N+1+7+14+2 \\ &= 3N+24 \end{aligned}$$

N	$N+1$	$N+2$
$N+7$	$(N+7)+1$	$(N+7)+2$
$N+14$	$(N+14)+1$	$(N+14)+2$

$$\begin{aligned} N+1+(N+7)+2+(N+14) &= 3N+1+7+2+14 \\ &= 3N+24 \end{aligned}$$

N	$N+1$	$N+2$
$N+7$	$(N+7)+1$	$(N+7)+2$
$N+14$	$(N+14)+1$	$(N+14)+2$

$$N+(N+7)+2+(N+14)+1=3N+7+2+14+1$$
$$=3N+24$$

N	$N+1$	$N+2$
$N+7$	$(N+7)+1$	$(N+7)+2$
$N+14$	$(N+14)+1$	$(N+14)+2$

$$N+2+(N+7)+1+(N+14)=3N+2+7+1+14$$
$$=3N+24$$

N	$N+1$	$N+2$
$N+7$	$(N+7)+1$	$(N+7)+2$
$N+14$	$(N+14)+1$	$(N+14)+2$

$$N+2+(N+7)+(N+14)+1=3N+2+7+14+1$$
$$=3N+24$$

The sum of the three numbers is $3N+24$. This is exactly 3 times the middle number.

Now it's your turn to work with calendar math.

3. Turn to pages 62 and 63 in your textbook. Do questions 1, 2, and 4 of "Put into Practice."

Check your answers on page 79 in the Appendix.

Turn to the Section 1 Assignment in Assignment Booklet 2A. Answer questions 5 to 9.

When you are finished, submit Assignment Booklet 2A to your teacher to be marked.

CONCLUSION

In this section you worked with patterns and puzzles and strengthened your pattern-recognition skills. You also learned how to graph pattern relations using ordered pairs. You made magic squares and decoded number tricks. You also saw how calendar pages can be used to develop number tricks.

Patterns are an important part of mathematics. It is important to be able to recognize and analyze them. Other patterns, such as the ones found in tartans and graphic designs, are also important—they might have historical importance and they are beautiful to look at.

SECTION 2

Equations and Graphs

The night sky has been a source of wonder for centuries. The planets of the solar system were thought to wander among the stars. Explanations for this apparent wandering ranged from the unbelievably complicated to the ridiculously simple. Careful long-term observations of the planets' positions by Tycho Brahe helped support the modern explanation of planetary motion. Johannes Kepler spent nearly a decade studying Brahe's data and unlocked the patterns hidden in it. Kepler determined three laws of planetary motion, which can be written as algebraic equations.

In this section you will be looking for ways to translate problems into equations. Then, you will use these equations to solve the problems. Don't worry though. You won't be dealing with any problems or equations nearly as complex as those for planetary motion!

LESSON 1

Solving One-Step Equations

Today you will study how to solve one-step equations.

When there is a difficult task ahead, it's very easy to put it off. These climbers have a tough climb ahead of them. Nevertheless, they have made the very difficult first steps. The rest of the climb is just one determined step after another.

Solving equations in mathematics is very much like that. First, you take a chance and start the process. Then you do a little more. Before you know it, the work is done. In this lesson you will be solving equations by taking that first step.

Turn to pages 212 and 213 in your textbook. Work through Example 1 carefully. You will be shown four ways to solve an equation. This example focuses on equations, such as $x-5=10$, where only one addition is needed to find the answer.

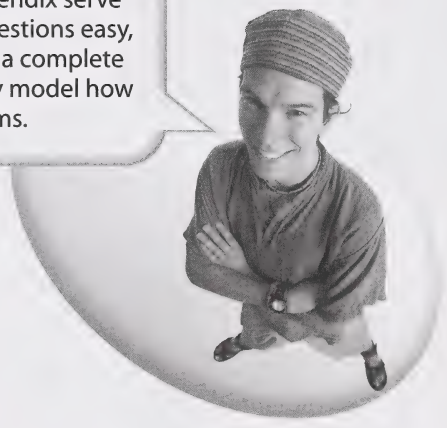
In this example and throughout this lesson, you will be using algebra tiles. Remove the page of cut-out algebra tiles from the Appendix. Cut them out and put them in an envelope. Keep the envelope in your Student Module Booklet so that you can access the algebra tiles as you work.

1. Solve each of the following equations.

a. $x-5=10$ **b.** $x-9=3$

c. $x-1=3$ **d.** $x-250=250$

Remember that the solutions in the Appendix serve as good examples. Even if you find the questions easy, study the solutions. They show you what a complete and correct solution should look like. They model how you should solve similar problems.



Turn to page 214 in your textbook. Work through Example 2 carefully. This example looks at equations, like $x+5=10$, where one subtraction is needed for the solution.

2. Solve each of the following equations.

a. $x+5=10$ **b.** $x+9=13$

c. $x+1=3$ **d.** $x+250=350$

Check your answers on pages 80 to 82 in the Appendix.

Turn to page 215 in your textbook. Work through Example 3 carefully. This example looks at equations, like $5x=10$, where one division is needed for the solution.

3. Solve each of the following equations.

a. $5x=10$ **b.** $9x=18$

c. $3x=12$ **d.** $250x=500$

Turn to pages 216 and 217 in your textbook. Work through Example 4 carefully. This example looks at equations, like $\frac{x}{5} = 10$, where one multiplication is needed for the solution.

4. Solve each of the following equations.

a. $\frac{x}{5} = 10$

b. $\frac{x}{9} = 3$

c. $\frac{x}{3} = 3$

d. $\frac{x}{250} = 4$

Check your answers on pages 83 to 86 in the Appendix.

5. Turn to pages 217 and 218 in your textbook. Answer questions 1, 3, 4, 5, and 6 of "Put into Practice." These questions will give you lots of practice with solving one-step equations.

Check your answers on pages 86 to 89 in the Appendix.

Turn to the Section 2 Assignment in Assignment Booklet 2B. Answer question 1.

LESSON 2

One-Step Equations—Problem Solving

Today you will investigate how to solve problems using one-step equations.

In medieval times, castle walls were used to keep enemies out. To an attacker, the wall was a major obstacle. Initially, all the attacker's resources would be put into getting over, under, or through the wall.

In mathematics, there are problems that can be solved by getting rid of the one big thing in the way. In this lesson you will learn how to recognize and conquer such problems.

Turn to page 219 in your textbook. Work through Example 1 carefully. You will be introduced to the four-step problem-solving method.

1. Turn to page 220 in your textbook. Answer questions 1 to 4 of "Investigation."

Check your answers on page 89 in the Appendix.

The four steps shown in "Investigation" help break a problem into smaller parts. Each part is a manageable part of the problem to work on. Success in each part ensures that the solution simply falls into place!

Step 1: Decide what you have to find.
Decide what the variable should represent.

Step 2: Set up an equation.
Translate the question into algebraic form.

Step 3: Solve the equation.

Step 4: Write a sentence to answer the question.

Turn to page 221 in your textbook. Study Example 2 closely. Obviously, Bob made a mistake in solving the problem. Can you guess what Bob did that made the price so large?

Example

Write an equation for each of the following statements.

1. The number of oranges is 3 less than the number of bananas.
2. The cost of a meal is $1.75 more than the cost of a sandwich.
3. A hockey stick costs 11 times as much as a hockey puck.
4. Forty-four eggs cost $5.72.

Solution

1. Let O represent the number of oranges. Let B represent the number of bananas.

"number of oranges" is 3 less than "number of bananas"

$O = -3 + B$

$O = -3 + B$ or $O = B - 3$

2. Let M represent the cost of a meal. Let S represent the cost of a sandwich.

"cost of a meal" is \$1.75 more than "cost of a sandwich"

$M = \$1.75 + S$

$M = \$1.75 + S$ or $M = S + \$1.75$

3. Let H represent the price of a hockey stick. Let P represent the price of a hockey puck.

"price of hockey stick" costs 11 times "price of hockey puck"

$H = 11 \times P$

$H = 11 \times P$ or $H = 11P$

4. Let E represent the cost of an egg.

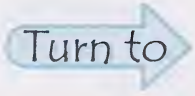
44 "eggs" cost \$5.72

$44E = \$5.72$

$44E = \$5.72$

2. Turn to pages 221 and 222 in your textbook. Answer questions 1, 2, 3, 7, 8, and 9 of "Put into Practice".

Check your answers on pages 89 and 91 in the Appendix.



the Section 2 Assignment in Assignment Booklet 2B. Answer question 2.

LESSON 3

Solving Two-Step Equations

Today you will study how to solve two-step equations.

This athlete has just finished a gruelling 3.86-km swim and is now competing in an equally gruelling 180.2-km bike race. Even then, it won't be over; there is still a 42.2-km marathon to run.

It takes tremendous stamina to complete an Ironman Triathlon. You have to be willing to carry on right to the end. In mathematics, it's sort of the same. You cannot quit after the first step; you have to keep on going. You will soon be seeing equations that aren't finished after the first step.

Turn to pages 225 and 226 in your textbook. Read the introduction to "Tutorial 4: Solving Two-Step Equations" closely. Then work through Example 1. You will see a sample two-step equation being solved.

Turn to page 227 in your textbook and work through Example 2. Note the two steps that are given on the right side of the page.

Example

Solve $\frac{x}{3} - 7 = 4$.

If $\frac{x}{3}$ was just y, you would have the simpler equation $y - 7 = 4$ to solve. Just think of $\frac{x}{3}$ as being equal to y. As you start to solve the equation, keep $\frac{x}{3}$ unchanged. To help you remember this, put $\frac{x}{3}$ in a box. You can think of the box as the variable y.

$$\boxed{\frac{x}{3}} - 7 = 4$$

$$\boxed{\frac{x}{3}} + 7 - 7 = 4 + 7 \qquad \text{Add 7 to each side.}$$

$$\boxed{\frac{x}{3}} = 11$$

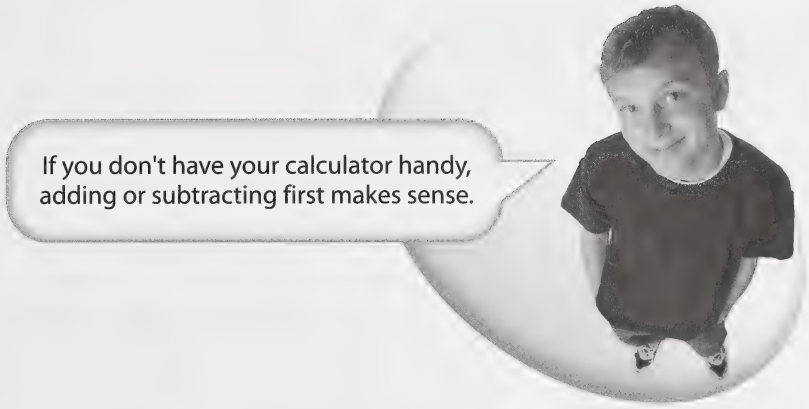
Now the first step is finished. When you remove the box, you get $\frac{x}{3} = 11$.

$$\frac{x}{3} = 11$$

$$3\left(\frac{x}{3}\right) = 3 \times 11 \qquad \text{Multiply each side by 3.}$$

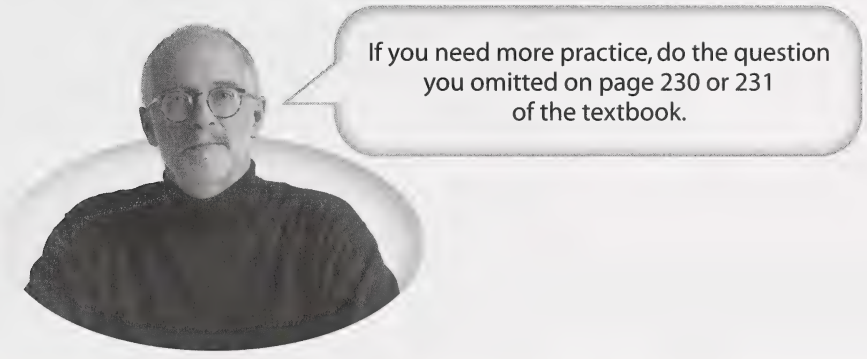
$$x = 33$$

Sometimes "thinking inside the box" can make a problem seem easier.



1. Turn to pages 227 to 229 in your textbook. Answer the question in "Investigation."

2. Turn to pages 229 to 231 in your textbook. Answer question 1 and either question 2 or question 3 of "Put into Practice." (**Note:** In question 2, change part r. to read $\frac{x}{2} - 3 = -2$ and part s. to read $2x + 5 = -9$. In question 3, change part 17 to read $\frac{c}{6} - 27 = -23$.)



Check your answers on pages 91 to 104 in the Appendix.

Turn to the Section 2 Assignment in Assignment Booklet 2B. Answer question 3.

LESSON 4

Verifying Answers to Equations

Today you will investigate how to **verify** solutions to equations.

When you're not well, you go to the doctor. Often the diagnosis is clear; sometimes it's not. In either case, you will likely have some tests done. These tests will be used to make sure that you are being treated properly. This verification of your doctor's initial diagnosis is an important part of modern medicine.

When you solve problems in mathematics, you need to check that your answers are correct. You will be verifying your answers to equations in this lesson.

Turn to page 223 in your textbook. Read the introduction to "Tutorial 3: Verifying Answers to One-Step Equations" closely. Then work through Example 1. You will see how to check a solution using the left side-right side method.

1. Turn to page 224 in your textbook. Answer the question in "Investigation."

Check your answers on page 104 in the Appendix.

Example

Solve the equation $4x - 3 = 5$ and verify your solution.

Solution

$$4x - 3 = 5$$

$$4x - 3 + 3 = 5 + 3 \quad \text{Add 3 to each side.}$$

$$4x = 8$$

$$\frac{4x}{4} = \frac{8}{4} \quad \text{Divide each side by 4.}$$

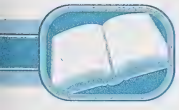
$$x = 2$$

In the verification, split the equation at the equal sign. Put everything on the left side of the equal sign in the Left Side column. Put everything on the right side of the equal sign in the Right Side column.

Verification

	Left Side	Right Side
	$4x - 3$	5
Substitute 2 for x.	$= 4(2) - 3$	
	$= 8 - 3$	
	$= 5$	

Since the left side equals the right side, $x = 2$ is the solution for $4x - 3 = 5$.



2. Turn to page 224 in your textbook. Answer questions 1 and 2 of "Put into Practice."

Check your answers on pages 105 to 109 in the Appendix.

Example

$x = 12$ was suggested as the solution to $x + 3 = 15$. Use the left side-right side method to check if this is the correct answer.

Verification

	Left Side	Right Side
	$x + 3$	15
Substitute 12 for x.	$= 12 + 3$	
	$= 15$	

Since the left side equals the right side, $x = 12$ is the solution for $x + 3 = 15$.

Turn to pages 232 and 233 in your textbook. Read the introduction to "Tutorial 5: Verifying Answers to Two-Step Equations." Then work through Examples 1 and 2.

3. Turn to page 233 in your textbook. Answer question 1 of "Put into Practice".

Check your answers on pages 110 to 113 in the Appendix.

Turn to the Section 2 Assignment in Assignment Booklet 2B. Answer question 4.

LESSON 5

Two-Step Equations—Problem Solving

Today you will examine solving problems that give two-step equations.

Even though castle walls were meant to keep enemies out, attackers soon learned how to break through a wall effectively. So castle owners tried to make things harder for attackers. With a moat to cross before reaching the castle wall, it was trickier. Now the attacker had to overcome a couple of troublesome obstacles.

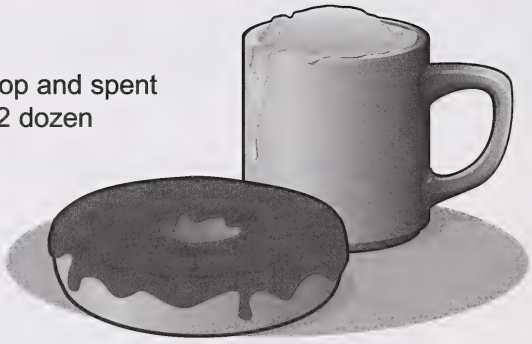
In mathematics, there are problems that have more than one thing in the way of a solution. You will soon see how to recognize and solve them.

Turn to page 234 in your textbook. Work through Example 1. You will use the same four steps used in solving one-step equation problems.

1. Turn to page 235 in your textbook. Do "Investigation."

Check your answers on page 113 in the Appendix.

Example



Elly's family stopped at a doughnut shop and spent $19.80, before GST. They purchased 2 dozen doughnuts and 6 cappuccinos. Each cappuccino cost $2.45. What did a dozen doughnuts cost?

You need to find the price of a dozen doughnuts.

Step 1: Let D be the price of a dozen doughnuts.

Step 2: The sum of 2 times the price of a dozen doughnuts plus the cost of 6 cappuccinos is $19.80.

$$2D + 6 \times \$2.45 = \$19.80$$

Step 3: Solve the equation.

$$\boxed{2D} + 6 \times \$2.45 = \$19.80 \quad \text{Find the total cost of the cappuccinos.}$$

$$\boxed{2D} + \$14.70 = \$19.80$$

$$\boxed{2D} + \$14.70 - \$14.70 = \$19.80 - \$14.70 \quad \text{Subtract \$14.70 from each side.}$$

$$\boxed{2D} = \$5.10$$

$$\frac{2D}{2} = \frac{\$5.10}{2} \quad \text{Divide each side by 2.}$$

$$D = \$2.55$$

2. Turn to pages 235 to 238 in your textbook. Do questions 1 to 8 and question 10 of "Put into Practice."

Check your answers on pages 114 to 118 in the Appendix.

Turn to the Section 2 Assignment in Assignment Booklet 2B. Answer question 5.

LESSON 6

Reading and Interpreting Graphs

Today you will read and interpret graphs.

Dividing a pie or circular cake can be a challenge. Should every piece be the same size? Would your brother accept a smaller piece than you? Is your dad trying to cut back on sweets? Do you need more energy for basketball practice? You have to know all of these things as you are making the cuts.

Most of us have had lots of practice judging whether one piece of pie is larger than another. You can see why dividing circles is a good technique for showing information about size. You'll be reading and making **pie graphs** in this lesson.

Turn to page 96 in your textbook. Read the introduction to "Tutorial 1: Reading and Interpreting Graphs."

1. Turn to pages 96 to 98 in your textbook. Do questions 2 to 5 of "Investigation."

Check your answers on pages 118 to 120 in the Appendix.

Example

Joe returned from a band trip to Kelowna. He paid $75 for the bus trip. He spent $50 on food, and lodging expenses totalled $45. Souvenirs cost $35 and entertainment was $25. Use technology to make a pie graph showing Joe's expenses.

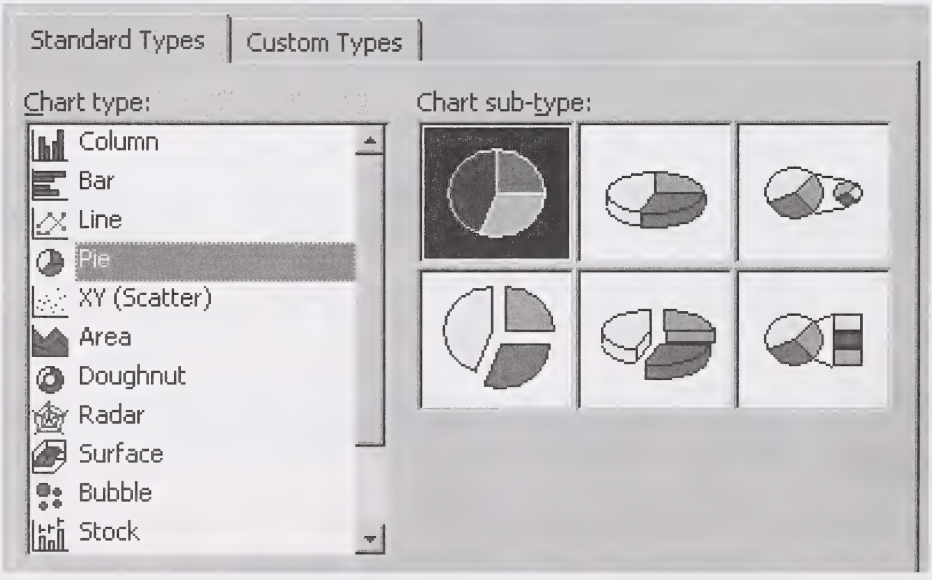
Start by entering the types of expenses in column A and the costs in column B of a spreadsheet.

	A	B
1	Bus	$75
2	Food	$50
3	Lodging	$45
4	Souvenirs	$35
5	Entertainment	$25

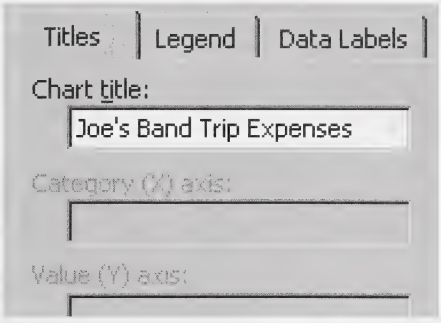
	A	B
1	Bus	$75
2	Food	$50
3	Lodging	$45
4	Souvenirs	$35
5	Entertainment	$25

Select the items you entered by clicking in cell A1 and dragging down to cell B5.

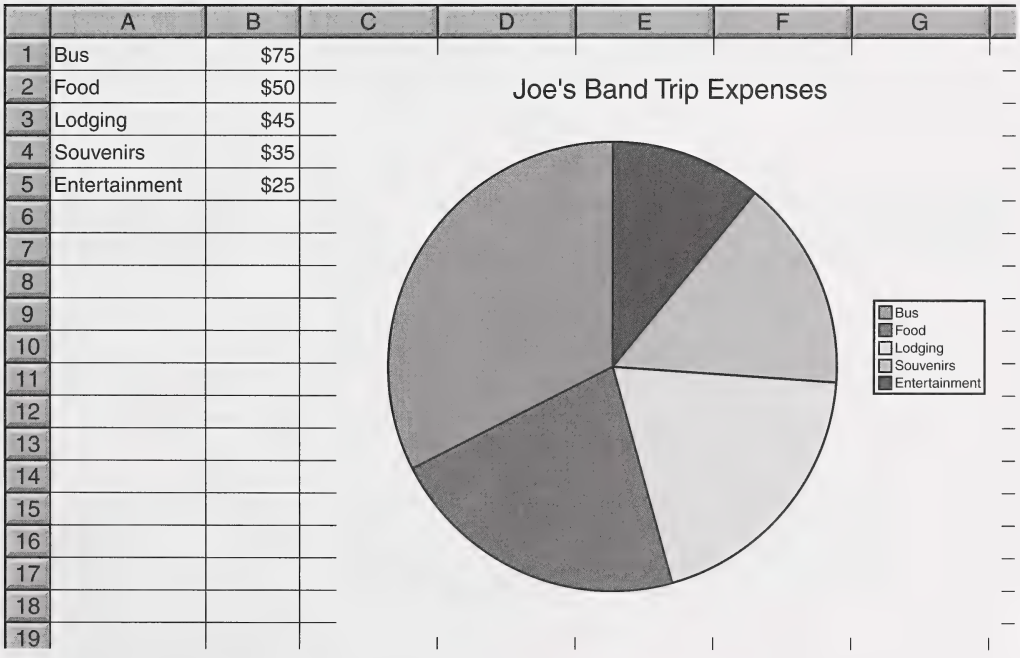
Start the chart wizard and select the pie chart type.



Click the Next button twice. Type in the title of the graph and then click the Finish button.



You will then have the following table and chart on your spreadsheet screen.



Example

Elease and José are having a party. They spent $50 on decorations, $110 on refreshments, $36 on entertainment, and $44 to rent extra tables and chairs. Make a hand-drawn pie chart to show their party expenses.

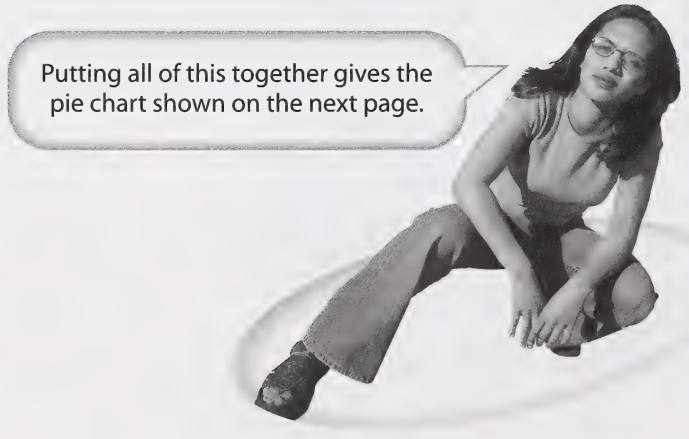
- Start by finding how much they spent in total.

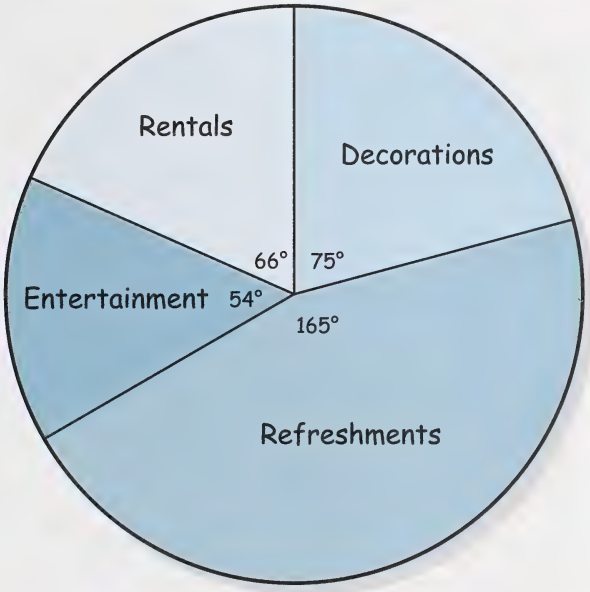
  $\$50 + \$110 + \$36 + \$44 = \$240$

- Now, choose a title for the pie chart. You might choose "Elease and José's Party Expenses."
- Now, draw a circle that is an appropriate size. You might make your circle about 10 cm in diameter.
- Now, cut up the circle. The four pieces have to be the right size.

You'll remember from earlier grades that circles measure 360°. You find the size of each part using this fact.

- The decoration part should be $\frac{\$50}{\$240} \times 360° = 75°$.
- The refreshment part should be $\frac{\$110}{\$240} \times 360° = 165°$.
- The entertainment part should be $\frac{\$36}{\$240} \times 360° = 54°$.
- The rental part should be $\frac{\$44}{\$240} \times 360° = 66°$.





2. Turn to pages 99 to 112 in your textbook.

a. Copy and complete the following charts in your notebook using the information from questions 1.b., 1.d., and 1.e. of "Put into Practice."

1. b.

People Visiting the Library on Monday Morning	
Age Range in Years	**Number of People**
0–15	
16–30	
31–45	
46–60	
61–75	

1. d.

Boxes of Recycled Paper Collected	
School Grade	**Number of Boxes**
3	
4	
5	
6	

1. e.

Number of Breaths in 15-s Interval During a Walk to the Library	
Time Walking in Minutes	**Number of Breaths in 15 s**
1	4
2	
3	
4	
5	

b. Do questions 3, 4, 5, 8, 12, 13, 14.a. to e., and 15 of "Put into Practice."

Check your answers on pages 121 to 125 in the Appendix.

Turn to the Section 2 Assignment in Assignment Booklet 2B. Answer questions 6 to 9.

CONCLUSION

In this section you investigated how to solve one-step and two-step equations. You used one-step and two-step equations to solve problems. You applied the left side-right side method of verifying solutions to equations. You have also read and interpreted bar graphs, pie graphs, line graphs, and pictographs.

A clear winter night sky, far from the glare of city lights, is filled with stars. Did you know that the light from the stars in distant galaxies in the outer reaches of the universe has been travelling towards Earth for billions of years? Astronomers observing these galaxies are really looking backwards in time. In order to investigate the origins of the universe, scientists must be good mathematicians. Some of the tools of mathematics they use daily in their work are the techniques of equation solving you studied in this section.

MODULE SUMMARY

In Section 1 you explored patterns and puzzles. You learned to recognize patterns, extend patterns, and express patterns in an algebraic form. You learned to graph patterns as points and when the pattern graph can be a line. You used your analytical skills to understand and solve numerical puzzles.

In Section 2 you explored equations and graphs. You learned to solve and verify one- and two-step equations and how to apply these types of equations in problem-solving situations. You learned to read and interpret bar graphs, line graphs, and pie graphs as well as histograms.

The study of patterns and equations is the essence of mathematics. One of the definitions of mathematics is the search for patterns in the world around you. The use of patterns in design and architecture is simply applied mathematics! If you have watched workers laying a patterned walk or drive, you have watched mathematics at work!

Turn to Assignment Booklet 2B and complete the Final Module Assignment.

When you are done, submit Assignment Booklet 2B to your teacher to be marked.

REVIEW

This Review will help you apply what you learned in Module 2 and prepare for the Final Test. Read over the skills checklist for this module. Use this list to guide your study and to help you decide how much of the Review you should complete.

Skills Checklist

Patterns

- ❑ Recognize a pattern.
- ❑ Visualize a pattern.
- ❑ Write an equation for a pattern.
- ❑ Graph a pattern.
- ❑ Determine if a graph should be made of points or a solid line.

Puzzles

- ❑ Describe and build magic squares.
- ❑ Solve and explain number tricks.
- ❑ Describe calendar number puzzles.

Graphs

- ❑ Read and interpret graphs.

Equations

- ❑ Solve one-step equations.
- ❑ Solve problems using one-step equations.
- ❑ Solve two-step equations.
- ❑ Solve problems using two-step equations.
- ❑ Verify solutions to equations.

If you need additional work to master the material in this module, work through the following lessons on the CD-ROM that accompanies your textbook:

- "Lesson 4: Patterns"
- "Lesson 5: Graphing Patterns"
- "Lesson 6: Understanding Graphs"
- "Lesson 13: Equations"

1. Turn to pages 65 to 67 in your textbook. Answer questions 5.a., 5.b., 6, 8, and 11.

2. a. Copy and complete this incomplete magic square in your notebook.

		17
	14	
	26	5

b. What is the magic sum for this magic square?

	30	

3. Explain why the following number trick always gives an answer of 4.

Pick a number. Double the number and add 3. Double your result and add 10. Divide the result by 4 and then subtract your original number.

4. Turn to pages 115 and 116 of your textbook. Answer questions 10.a., 10.b., and 11.

5. a. Solve the following equations.

i. $w - 6 = 27$ **ii.** $c + 3 = -11$

iii. $\frac{x}{17} = 3$ **iv.** $4y = 64$

b. Solve the following equations.

i. $3q - 3 = 9$ **ii.** $8r + 11 = 59$

iii. $\frac{m}{5} + 7 = 9$ **iv.** $\frac{k}{12} - 6 = -1$

c. Verify the given sloutions. If the solution does not verify, find the correct solution and verify it.

i. $q = 17$ as a solution to $4q = 68$

ii. $s = 3$ as a solution to $8s - 5 = 27$

6. Turn to pages 286 and 287 of your textbook. Answer questions 7 and 10.

Check your answers on pages 125 to 131 in the Appendix.

MATHEMATICS 14

Appendix

GLOSSARY
ANSWER KEY
IMAGE CREDITS
LEARNING AIDS

Glossary

equation: a mathematical statement of the equality of two quantities

For example, $3x+5=14$ and $C=2\pi r$ are equations.

linear relationship: a relationship that places all points on a straight line

one-step equation: an equation that can be solved using one mathematical operation

For example, $x+5=14$, $x-5=14$, $5x=25$, and $\frac{x}{5}=3$ are one-step equations.

pattern: an arrangement, design, or repeated occurrence

perfect square: a number that is the square of a natural number

pie graph: a graph that uses a circle divided into parts to show the relative size of various items

substitution: replacing a variable with a value

two-step equation: an equation that can be solved using two mathematical operations

For example, $3x+5=14$ and $\frac{x}{6}-2=-1$ are two-step equations.

variable: a letter representing a value

verify: to check or test that an answer is correct

Answer Key

Section 1: Patterns and Puzzles

Lesson 1: Recognizing Patterns

1. Textbook, pages 39 and 40, "Investigation," questions a. through e.

a. When the number of plant stands increases, the number of legs increases. Each new plant stand adds 4 new legs.

b. There are 12 legs on 3 plant stands. Four plant stands would have $12+4=16$ legs. Five plant stands would have $16+4=20$ legs.

c. There are 4 times as many legs as there are plant stands. This can be written as an equation. The equation given in the hint is $L=4P$.

d. Substitute 72 for P in the equation.

Explain what to do.

$$\begin{aligned} L &= 4P \\ &= 4 \times 72 \\ &= 288 \end{aligned}$$

Carry out the needed calculations.

There would be 288 legs on 72 plant stands.

Write a concluding statement.

e. Substitute 244 for L in the equation.

Explain what to do.

$$\begin{aligned} L &= 4P \\ 244 &= 4 \times P \\ \frac{244}{4} &= \frac{4 \times P}{4} \quad \text{Divide each side by 4.} \\ 61 &= P \end{aligned}$$

Carry out the needed calculations.

Sixty-one plant stands have 244 legs.

Write a concluding statement.

2. Textbook, pages 41 to 44, "Put into Practice," questions 1 to 5

1. a.

Squares in Tail (t)	Squares in All (a)
1	10
2	11
3	12
4	13
5	14
6	15
7	16
8	17
9	18
10	19

b. There are 9 more "squares in all" than there are "squares in the tail."

c. "Squares in all" is "squares in tail" plus 9.

$$a = t + 9$$

d. i. Substitute 12 for t. Explain what to do.

$$a = t + 9$$
$$a = 12 + 9$$
$$a = 21$$

Carry out the needed calculations.

Twelve tail squares means 21 squares in all. Write a concluding statement.

ii. Substitute 23 for t.

$$a = t + 9$$
$$a = 23 + 9$$
$$a = 32$$

Twenty-three tail squares means 32 squares in all.

iii. Substitute 40 for t.

$$a = t + 9$$
$$a = 40 + 9$$
$$a = 49$$

Forty tail squares means 49 squares in all.

e. i. Substitute 113 for a.

$$a = t + 9$$
$$113 = t + 9$$
$$113 - 9 = t + 9 - 9$$
$$104 = t$$

One hundred thirteen squares means 104 tail squares.

ii. Substitute 45 for a.

$$a = t + 9$$
$$45 = t + 9$$
$$45 - 9 = t + 9 - 9$$
$$36 = t$$

Forty-five squares in all means 36 tail squares.

iii. Substitute 172 for a.

$$a = t + 9$$
$$172 = t + 9$$
$$172 - 9 = t + 9 - 9$$
$$163 = t$$

One hundred seventy-two squares in all means 163 tail squares.

2. a.

Number of Tables (t)	Number of People Seated (p)
1	4
2	6
3	8
4	10
5	12
6	14
7	16
8	18

b. Each time a new table is added, the number of people increases by 2. There will be a "multiply by 2" in the pattern. Try 2 times the number of tables. Test this theory with 1 table. This would give 2 people for 1 table. This is 2 people less than needed. That means 2 people must be added. The "number of people seated" is twice the "number of tables" plus 2. This seems to work for each number of tables.

c. "Number of people seated" is twice the "number of tables" plus 2.

$p = 2 \times t + 2$ or $p = 2t + 2$

d. i. Substitute 15 for t.

$p = 2t + 2$
$p = (2 \times 15) + 2$
$p = 30 + 2$
$p = 32$

Thirty-two people can sit at 15 tables.

ii. Substitute 37 for t.

$p = 2t + 2$
$p = (2 \times 37) + 2$
$p = 74 + 2$
$p = 76$

Seventy-six people can sit at 37 tables.

iii. Substitute 112 for t.

$$p = 2t + 2$$
$$p = (2 \times 112) + 2$$
$$p = 224 + 2$$
$$p = 226$$

Two hundred twenty-six people can sit at 112 tables.

e. i. Substitute 48 for p.

$$p = 2t + 2$$
$$48 = 2t + 2$$
$$48 - 2 = 2t + 2 - 2$$
$$\frac{46}{2} = \frac{2t}{2}$$
$$23 = t$$

Twenty-three tables are needed for 48 people.

ii. Substitute 30 for p.

$$p = 2t + 2$$
$$30 = 2t + 2$$
$$30 - 2 = 2t + 2 - 2$$
$$\frac{28}{2} = \frac{2t}{2}$$
$$14 = t$$

Fourteen tables are needed for 30 people.

iii. Substitute 94 for p.

$$p = 2t + 2$$
$$94 = 2t + 2$$
$$94 - 2 = 2t + 2 - 2$$
$$\frac{92}{2} = \frac{2t}{2}$$
$$46 = t$$

Forty-six tables are needed for 94 people.

3. a.

Number of Triangles (t)	Number of Toothpicks (n)
1	3
2	5
3	7
4	9
5	11
6	13

b. Each time a new triangle is created, 2 new toothpicks are added. A "multiply by 2" is involved. Try 2 times the number of triangles. Test this theory with 1 triangle. This gives 2 toothpicks for 1 triangle. This is 1 toothpick less than needed. Add one toothpick. The "number of toothpicks" is twice the "number of triangles" plus 1. This seems to work for each number of triangles.

c. "Number of toothpicks" is twice the "number of triangles" plus 1.

$n = 2 \times t + 1$ or $n = 2t + 1$

d. i. Substitute 19 for t.

$n = 2 \times t + 1$
$n = (2 \times 19) + 1$
$n = 38 + 1$
$n = 39$

Nineteen triangles require 39 toothpicks.

ii. Substitute 34 for t.

$n = 2 \times t + 1$
$n = (2 \times 34) + 1$
$n = 68 + 1$
$n = 69$

Thirty-four triangles require 69 toothpicks.

iii. Substitute 213 for t.

$n = 2 \times t + 1$
$n = (2 \times 213) + 1$
$n = 426 + 1$
$n = 427$

Two hundred thirteen triangles require 427 toothpicks.

e. i. Substitute 57 for n.

$$n = 2 \times t + 1$$
$$57 = 2 \times t + 1$$
$$57 - 1 = 2 \times t + 1 - 1$$
$$\frac{56}{2} = \frac{2 \times t}{2}$$
$$28 = t$$

Fifty-seven toothpicks make 28 triangles.

ii. Substitute 191 for n.

$$n = 2 \times t + 1$$
$$191 = 2 \times t + 1$$
$$191 - 1 = 2 \times t + 1 - 1$$
$$\frac{190}{2} = \frac{2 \times t}{2}$$
$$95 = t$$

One hundred ninety-one toothpicks make 95 triangles.

iii. Substitute 75 for n.

$$n = 2 \times t + 1$$
$$75 = 2 \times t + 1$$
$$75 - 1 = 2 \times t + 1 - 1$$
$$\frac{74}{2} = \frac{2 \times t}{2}$$
$$37 = t$$

Seventy-five toothpicks make 37 triangles.

4. a.

Number of Steps (s)	Number of Bricks (b)
1	1
2	3
3	6
4	10
5	15
6	21
7	28

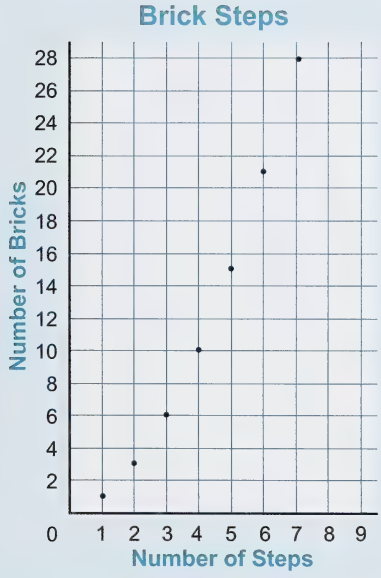
b. Each step adds as many new bricks as the number of the step. The total number of bricks is the sum of the numbers up to the number of steps. (You weren't asked for this, but $b = \frac{s(s+1)}{2}$ is an equation that calculates this sum.)

c. Seventeen steps will take $17+16+15+14+\ \ldots\ +2+1=153$ bricks.

$$b = \frac{17 \times 18}{2}$$
$$= \frac{306}{2}$$
$$= 153$$

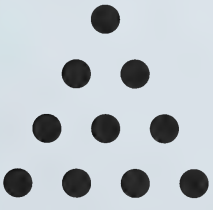
The equation gives the same answer.

d.

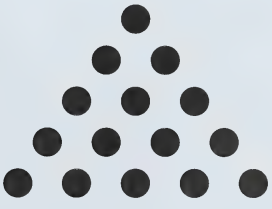


The points are not on a straight line.

5. a. The fourth triangular number has 10 dots.



b. The fifth triangular number has 15 dots.



Lesson 2: Graphing Patterns

1. Textbook, page 45, "Put into Practice," questions a. and b.

a. i. Point A has the co-ordinates $(1, 1)$. (It is 1 unit right and 1 unit up.)

ii. Point F has the co-ordinates $(10, 0)$. (It is 10 units right and 0 units up.)

iii. Point K has the co-ordinates $(3, 1)$. (It is 3 units right and 1 unit up.)

iv. Point G has the co-ordinates $(8, 9)$. (It is 8 units right and 9 units up.)

v. Point C has the co-ordinates $(3, 6)$. (It is 3 units right and 6 units up.)

b. i. (3, 1) has the letter K.
ii. (5, 8) has the letter B.
iii. (2, 5) has the letter J.
iv. (9, 7) has the letter D.
v. (10, 0) has the letter F.
vi. (0, 0) has the letter L.

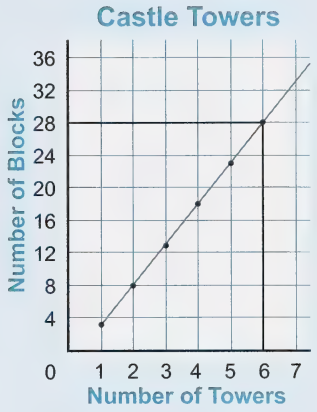
2. Textbook, page 46, "Investigation"

Number of Towers	Number of Blocks	Ordered Pairs
1	3	$(1, 3)$
2	8	$(2, 8)$
3	13	$(3, 13)$
4	18	$(4, 18)$
5	23	$(5, 23)$
6	28	$(6, 28)$

Use the graph to predict how many blocks are needed for a 6-tower castle.

- Join the dots with a line and extend the line.
- Move up from the 6 on the *x*-axis.
- Turn left when the extended line is met.
- Move to the *y*-axis.
- Read the value of 28 blocks.

The graph predicts that 28 blocks are needed for a 6-tower castle.



3. Textbook, pages 47 and 48, "Put into Practice," questions 1, 2, and 3

1. a.

Number of Towers	Number of Blocks	Ordered Pairs
1	3	$(1, 3)$
2	7	$(2, 7)$
3	11	$(3, 11)$
4	15	$(4, 15)$
5	19	$(5, 19)$
6	23	$(6, 23)$

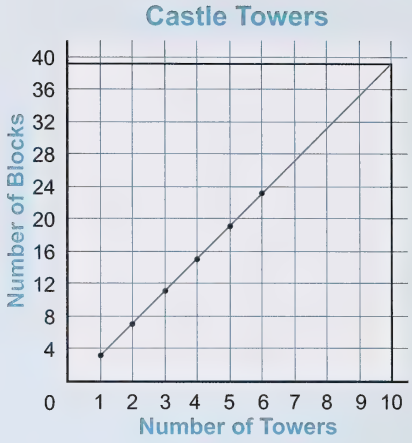
b. The ordered pairs are $(1, 3), (2, 7), (3, 11), (4, 15), (5, 19)$, and $(6, 23)$.

c. Each additional tower adds 4 blocks. There should be a factor of 4. Try 4 times the number of towers. Test this theory for 1 tower. This gives 4 blocks for 1 tower. This is one block too many. Subtract one block. The number of blocks is 4 times "the number of towers" minus 1. This seems to work for any number of towers.

d. number of blocks = 4 times the number of towers -1

$B = 4 \times T - 1$ or $B = 4T - 1$

e. A straight line would pass through the points.



f. To find the number of blocks needed for 10 towers, do the following:

- Extend the line through the points.
- Start at 10 on the *x*-axis.
- Move up to the graph line.
- Turn toward the *y*-axis.
- Move to the *y*-axis.
- Read the value of 39.

Thirty-nine blocks are needed for 10 towers.

g. Substitute 10 for T.

$$B = 4 \times T - 1$$
$$B = (4 \times 10) - 1$$
$$B = 40 - 1$$
$$B = 39$$

It takes 39 blocks to build a 10-tower castle.

2.

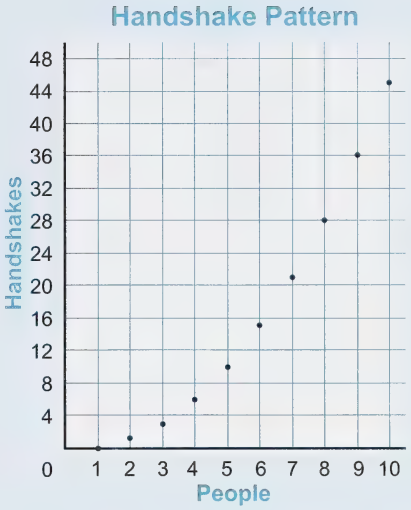
Number of Towers	Number of Blocks
1	4
2	12
3	20
4	28
5	36
6	44

There would be 5 towers in a 36-block wall.

3. a.

People	Shakes	Ordered Pairs
1	0	(1, 0)
2	1	(2, 1)
3	3	(3, 3)
4	6	(4, 6)
5	10	(5, 10)
6	15	(6, 15)
7	21	(7, 21)
8	28	(8, 28)
9	36	(9, 36)
10	45	(10, 45)

b.



c. This is not a linear relationship. No straight line passes through all the points.

Lesson 3: Points or Lines?

1. Textbook, pages 49 and 50, "Investigation"

1. a. For each new customer, Mitchell gets \$8.50 more. Mitchell is paid \$20.00 plus \$8.50 times the number of customers he signs up.

$$e = \$20.00 + \$8.50 \times c$$

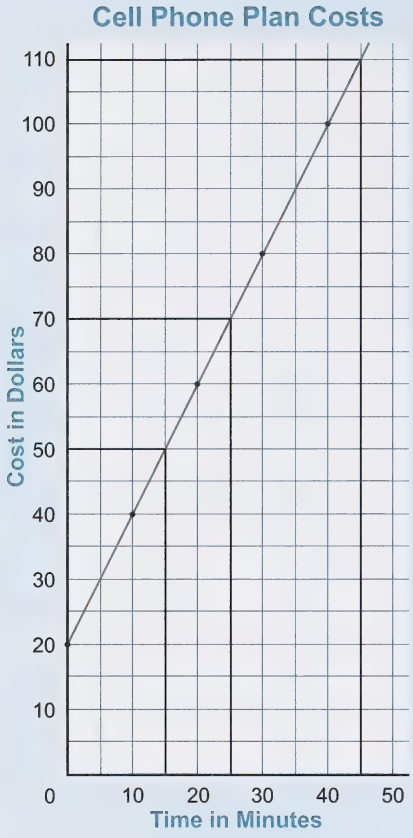
b. The table follows and the graph is given in the textbook.

c	0	1	2	3	4	5	6
e	\$20.00	\$28.50	\$37.00	\$45.50	\$54.00	\$62.50	\$71.00

c. Do not join the points on the graph. People are added as individuals. Only a whole number of people can be signed up.

2. Textbook, pages 51 and 52, "Put into Practice," questions 1 to 4

1. a. No, you should not join the points. Carol's bill will either be based on the time rounded upward to the nearest minute or charges rounded to the nearest cent. In either case, a whole number of minutes or a whole number of cents means you should not join the points.



The grey line is not part of the graph. It was added to let the answers for parts b. to d. to be found.

b. Carol would pay $70 for 25 min of use. See the line going up from 25 min and then across to $70.

c. Carol would pay $110 for 45 min of use. See the line going up from 45 min and then over to $110.

d. A $50 bill would be for 15 min of use. See the line going over from $50 and then down to 15 min.

e. **i.** She pays by the minute or part of a minute plus a fixed monthly charge of $20.
ii. She pays $20 plus $2 per minute of calls.

2. a. The mass of one melon is 3 kg.

$$\begin{aligned} \text{total mass} &= \text{mass of case} + \text{mass of melons} \\ 35 \text{ kg} &= 5 \text{ kg} + 30 \text{ kg} \\ &= 5 \text{ kg} + 10 \times 3 \text{ kg} \end{aligned}$$

There are 10 melons, so the melon mass must be broken into 10 smaller masses.

b. The mass of the crate of fruit is 5 kg plus the mass of the melons. The mass, M, of the crate of fruit is 5 kg plus the number, n, of melons times the weight of a melon.

$$M = 5 + 3 \times n \text{ or } M = 3n + 5$$

c.

Number of Melons	Mass (kg)
0	5
1	8
2	11
10	35

Ordered Pairs

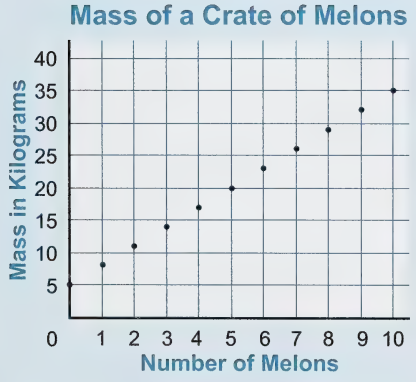
(0, 5)

(1, 8)

(2, 11)

(10, 35)

d.



e. You should not join the points. The mass increases one melon at a time.

3. a. Nigel's salary is \$15 plus \$3 per souvenir sold.

Nigel's salary is \$15 plus \$3 times the number of souvenirs sold.

$$W = \$15 + \$3 \times S \text{ or } W = 3S + 15$$

b. The amount for each souvenir would be decreased. Nigel would now get only half as much for each souvenir sold. He would get $\$3.00 \div 2 = \$1.50$ per souvenir. The new equation would be $W = \$15.00 + \$1.50 \times S$.

4. There are $20 + 12 = 32$ adult passengers on the two trains.

There are $16 + 24 = 40$ student passengers on the two trains.

Total fare collected is 32 adult fares plus 40 student fares. Total fare collected is 32 times $\$a$ plus 40 times $\$K$.

$$T = 32 \times \$a + 40 \times \$K$$
or $T = \$32a + \$40K$
or $T = \$(32a + 40K)$

Lesson 4: Magic Squares

1. Textbook, page 53, "Investigation," questions 1 and 2

1. a. The sums of the columns are shown.

8	1	6
3	5	7
4	9	2
15	15	15

b. The sums of the rows are shown.

8	1	6	15
3	5	7	15
4	9	2	15

c. The sums of the diagonals are shown.

			15
8	1	6	
3	5	7	
4	9	2	
			15

d. All of the sums are the same.

e. The magic sum is **3** times the number in the middle cell.

2. a. This is a magic square. Each column, row, and diagonal adds to 65.

b. The magic sum is **5** times the number in the middle cell.

2. Textbook, page 54, "Put into Practice," questions 1 and 2

1. a. The first nine numbers are 2, 5, 8, 11, 14, 17, 20, 23, and 26.

b.

23 (8)	2 (1)	17 (6)
8 (3)	14 (5)	20 (7)
11 (4)	26 (9)	5 (2)

The new magic square is shown. Place 2 in square 1, 5 in square 2, and so on, until all terms have been put in the correct square.

c. This is a magic square. Each column, row, and diagonal adds to 42.

d. The magic sum is 3 times the number in the middle cell.

$$42 = 3 \times 14$$

2. Answers will vary. A sample answer is given.

a. Use a start number of 11. Use an increment of 5. The new sequence will be 11, 16, 21, 26, 31, 36, 41, 46, and 51.

b.

46 (8)	11 (1)	36 (6)
21 (3)	31 (5)	41 (7)
26 (4)	51 (9)	16 (2)

A square using these numbers is shown. Starting with 11 in square 1, the succeeding terms are placed in the squares.

c. It is a magic square. Each column, row, and diagonal adds to 93. The magic sum is 3 times the number in the middle cell.

$$93 = 3 \times 31$$

3. Textbook, pages 56 to 58, "Put into Practice," questions 3, 4, 5, 7, 8, 9, 10.a., and 11

3. a.

$n+7$	n	$n+5$
$n+2$	$n+4$	$n+6$
$n+3$	$n+8$	$n+1$

b.

Column Sums

$n+7$	n	$n+5$
$n+2$	$n+4$	$n+6$
$n+3$	$n+8$	$n+1$
$3n+12$	$3n+12$	$3n+12$

Row Sums

$n+7$	$n+2$	$n+3$
n	$n+4$	$n+8$
$n+5$	$n+6$	$n+1$
$3n+12$	$3n+12$	$3n+12$

Diagonal Sums

$n+7$	$n+3$
$n+4$	$n+4$
$n+1$	$n+5$
$3n+12$	$3n+12$

4.

26	51-26-20 5 2	51-8-23 20 1
51-17-23 11 3	17	23
51-17-20 14 4	51-17-5 29 5	8

The sum of the diagonal is 51. All other rows and columns must be made to add up to 51. The calculation to find each other square is shown in grey at the top of the square. The numbers at the bottom of the square show the order in which the new values were calculated.

5.

72-4-29 39 5	4	72-24-19 29 1
72-19-33 14 6	24	72-9-29 34 4
19	72-4-24 44 2	72-19-44 9 3

Three times the middle cell is 72. All rows, diagonals, and columns must be made to add up to 72. The calculation to find each unknown square is shown in grey at the top of the square. The numbers at the bottom of the square show the order in which the new values were calculated.

7. This is not a magic addition square. Column 1 adds to 140. Column 2 adds to 273.

8. a. $128 \times 4 \times 8 = 4096$

b. $1 \times 16 \times 256 = 4096$

c. The products are the same.

d. This is a magic multiplication square.

Columns	Rows	Diagonals
$128 \times 4 \times 8 = 4096$	$128 \times 1 \times 32 = 4096$	$128 \times 16 \times 2 = 4096$
$1 \times 16 \times 256 = 4096$	$4 \times 16 \times 64 = 4096$	$8 \times 16 \times 32 = 4096$
$32 \times 64 \times 2 = 4096$	$8 \times 256 \times 2 = 4096$	

9. a. The nine numbers are 2, 6, 18, 54, 162, 486, 1458, 4374, and 13 122.

b. The magic multiplication square follows. Starting with 2 in square 1, add the succeeding terms to the rest of the squares in order.

4374 (8)	2 (1)	486 (6)
18 (3)	162 (5)	1458 (7)
54 (4)	13 722 (9)	6 (2)

c. The magic product is $54 \times 162 \times 486 = 4\ 251\ 528$.

10. Answers will vary. A sample answer is given.

Here are the steps to create a magic multiplication square:

- Pick a number to start with. (Try 5.)
- Pick a number to multiply by. (Try 3.)

- Make a sequence with 9 terms using the chosen numbers.

$$
\begin{aligned}
&5\\
5\times 3 &= 15\\
15\times 3 &= 45\\
45\times 3 &= 135\\
135\times 3 &= 405\\
405\times 3 &= 1215\\
1215\times 3 &= 3645\\
3645\times 3 &= 10\,935\\
10\,935\times 3 &= 32\,805
\end{aligned}
$$

The sequence is 5, 15, 45, 135, 405, 1215, 3645, 10 935, and 32 805.

- Place the terms in the appropriate squares (first term in cell 1, second term in cell 2, and so on).

8	1	6
3	5	7
4	9	2

10 935	5	1215
45	405	3645
135	32 805	15

11. The magic product is $531\,441 = 2187 \times 81 \times 3$. The unknown values were calculated in the order shown by the circled numbers after the equations.

2187	1	243
9	81	729
27	6561	3

$(531\,441 \div 243) \div 2187 = 1$ (3)

$(531\,441 \div 3) \div 729 = 243$ (2)

$(531\,441 \div 9) \div 81 = 729$ (1)

$(531\,441 \div 243) \div 81 = 27$ (4)

$(531\,441 \div 1) \div 81 = 6561$ (5)

Lesson 5: Number Tricks and Patterns

1. Textbook, page 59, "Investigation," questions a. to f.

Answers will vary. Sample answers using 43 and 11 follow.

a. $43 \times 3 = 129$ $\quad 11 \times 3 = 33$
b. $129 + 9 = 138$ $\quad 33 + 9 = 42$
c. $138 \div 3 = 46$ $\quad 42 \div 3 = 14$
d. $46 - 2 = 44$ $\quad 14 - 2 = 12$
e. $44 - 43 = 1$ $\quad 12 - 11 = 1$
f. All others should have an answer of 1 as well.

2. Textbook, page 61, "Put into Practice," questions 1, 2, and 3

1. Answers will vary. Sample answers using 17 and 2 follow.

a. $17 + 3 = 20$ $\quad 2 + 3 = 5$

b. $20 \times 2 = 40$ $\quad 5 \times 2 = 10$

c. $40 - 2 = 38$ $\quad 10 - 2 = 8$

d. $38 \div 2 = 19$ $\quad 8 \div 2 = 4$

e. $19 - 17 = 2$ $\quad 4 - 2 = 2$

f. All others should have an answer of 2 as well.

g. Let the number chosen be called x. The steps of the trick done algebraically follow.

- $x + 3$ Add 3.
- $(x + 3) \times 2 = 2x + 6$ Multiply by 2.
- $(2x + 6) - 2 = 2x + 4$ Subtract 2.
- $(2x + 4) \div 2 = x + 2$ Divide by 2.
- $(x + 2) - x = 2$ Subtract the original number.

2. Answers will vary. A sample solution using the number 7 follows.

Number	Algebra
7	X
$7+17=24$	$X+17$
$24\times2=48$	$(X+17)\times2=2X+34$
$48-4=44$	$2X+34-4=2X+30$
$44\times2=88$	$2X+(30\times2)=4X+60$
$88+20=108$	$4X+60+20=4X+80$
$108\div4=27$	$4X+(80\div4)=X+20$
$27-20=7$	$X+20-20=X$
$7-7=0$	$X-X=0$

All others should get 0 as well. The explanation is shown in the algebra column.

3. First find the perfect squares between 50 and 100. Since $7\times7=49$ and $10\times10=100$, the natural numbers that could make the needed perfect square are 8 and 9. Any other natural numbers would give values that are too small or too big. $8\times8=64$ and $9\times9=81$ are the only perfect squares between 50 and 100.

Second, find the sum of the digits of the perfect square.

$6+4=10$
$8+1=9$

Third, see which sum is a perfect square.

10 is not a perfect square.
$9=3\times3$ is a perfect square.

Eighty-one is a perfect square between 50 and 100. The sum of the digits of 81 is a perfect square.

Ms. Nastasiuk wanted her class to turn to page 81.

3. Textbook, pages 62 and 63, "Put into Practice," questions 1, 2, and 4

1. a. The vertical pattern is to add 7.

$8+7=15$ $9+7=16$

b. The diagonal sums are equal.

$8+16=24$ $15+9=24$

2. Answers will vary. General answers for the questions follow.

a. The same patterns will be found. The horizontal pattern is to add 1. The vertical pattern is to add 7. The diagonal sums are equal.

b. Examples like that shown in the text and in question 1 would be expected.

4. The sum of the diagonals will be $16\times3=48$. Or look at the calendar and add the diagonal elements.

8	9	10
15	16	17
22	23	24

The diagonal sums are $8+16+24=48$ and $22+16+10=48$.

Section 2: Equations and Graphs

Lesson 1: Solving One-step Equations

1. Methods may vary. You can use any of the four methods to solve these questions. Sample solutions using algebra tiles and the reverse operation are shown.

a.

$x - 5 = 10$

5 is subtracted from x.

You add 5 to each side.

$x - 5 = 10$

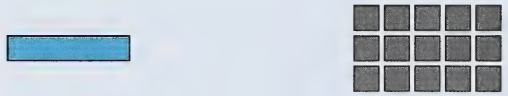
$x - 5 + 5 = 10 + 5$

$x = 15$

$x - 5 = 10$

$+5 \quad +5$

To get x alone, you need to put the 5 back.



$x = 15$

To keep the equation balanced, you must put 5 more on each side.

b.

$x - 9 = 3$

9 is subtracted from x.

You add 9 to each side.

$x - 9 = 3$

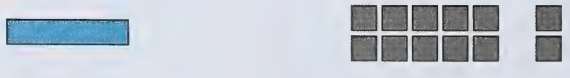
$x - 9 + 9 = 3 + 9$

$x = 12$

$x - 9 = 3$

$+9 \quad +9$

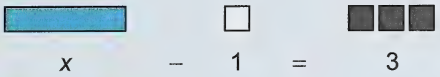
To get x alone, you need to put the 9 back.



$x = 12$

To keep the equation balanced, you must put 9 more on each side.

c.



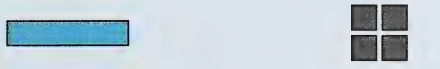
$x - 1 = 3$

1 is subtracted from x.

$x - 1 = 3$

$x \quad +1 \quad +1$

To get x alone, you need to put the 1 back.



$x = 4$

To keep the equation balanced, you must put 1 more on each side.

You add 1 to each side.

$$x - 1 = 3$$
$$x - 1 + 1 = 3 + 1$$
$$x = 4$$

d. The numbers are too large to use algebra tiles easily.

You add 250 to each side.

$$x - 250 = 250$$
$$x - 250 + 250 = 250 + 250$$
$$x = 500$$

2. You can use any of the four methods to solve these questions. Solutions using algebra tiles and the reverse operation are shown.

a.

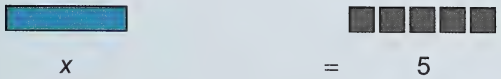
$x + 5 = 10$

5 has been added to x.

$x + 5 \quad -5$

$10 \quad -5$

You need to remove it.



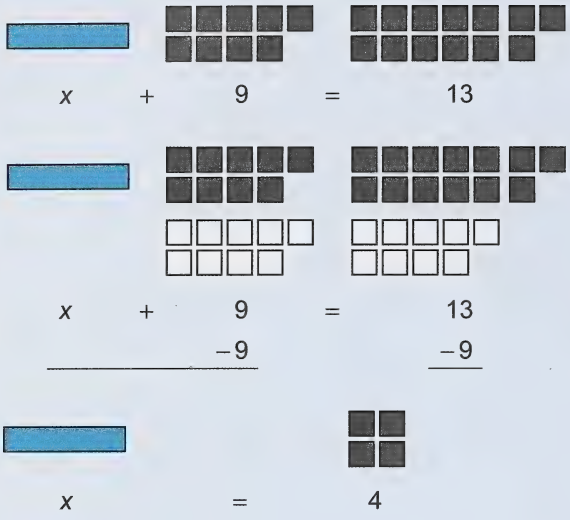
$x = 5$

To keep the equation balanced, you must take 5 from each side.

Subtract 5 from each side.

$$x + 5 = 10$$
$$x + 5 - 5 = 10 - 5$$
$$x = 5$$

b.



9 has been added to x.

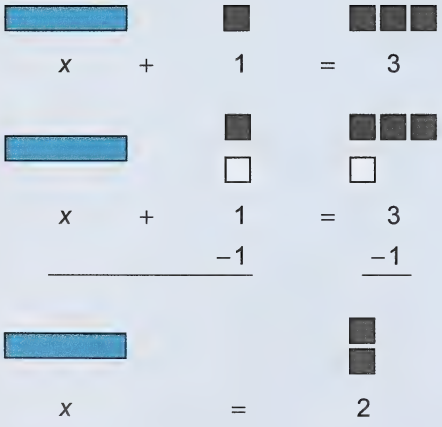
You need to remove it.

To keep the equation balanced, you must take 9 from each side.

Subtract 9 from each side.

$$x + 9 = 13$$
$$x + 9 - 9 = 13 - 9$$
$$x = 4$$

c.



1 has been added to x.

You need to remove it.

To keep the equation balanced, you must take 1 from each side.

Subtract 1 from each side.

$$x + 1 = 3$$
$$x + 1 - 1 = 3 - 1$$
$$x = 2$$

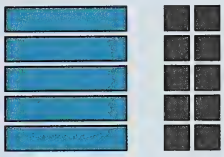
d. The numbers are too large to use algebra tiles easily.

Subtract 250 from each side.

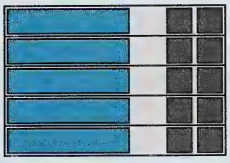
$$x + 250 = 350$$
$$x + 250 - 250 = 350 - 250$$
$$x = 100$$

3. You can use any of the four methods to solve these questions. Solutions using algebra tiles and the reverse operation are shown.

a.



$5x = 10$



$x = 2$

Divide each side by 5.

$$5x = 10$$

$$5x = \frac{10}{5}$$

$$x = 2$$

b.

$9x = 18$

$x = 2$

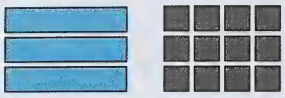
Divide each side by 9.

$$9x = 18$$

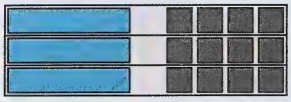
$$\frac{9x}{9} = \frac{18}{9}$$

$$x = 2$$

c.



$3x = 12$



$x = 4$

Divide each side by 3.

$$3x = 12$$
$$\frac{3x}{3} = \frac{12}{3}$$
$$x = 4$$

d. The numbers are too large to use algebra tiles easily.

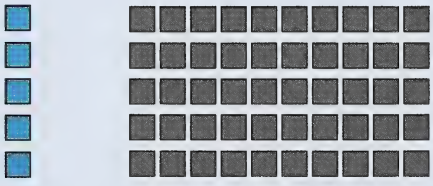
Divide each side by 250.

$$250x = 500$$
$$\frac{250x}{250} = \frac{500}{250}$$
$$x = 2$$

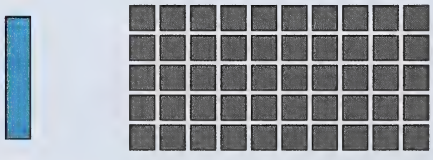
4. a.

x

$\frac{x}{5} = 10$



$5 \times \frac{x}{5} = 5 \times 10$

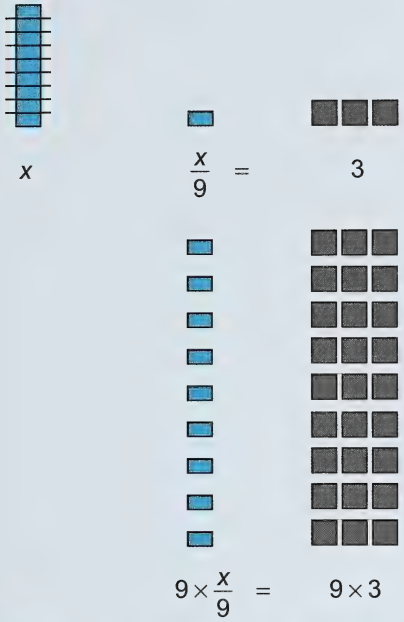


$x = 50$

Multiply each side by 5.

$$\frac{x}{5} = 10$$
$$5\left(\frac{x}{5}\right) = 5 \times 10$$
$$x = 50$$

b.



$x \quad \frac{x}{9} = 3$

$9 \times \frac{x}{9} = 9 \times 3$

$x = 27$

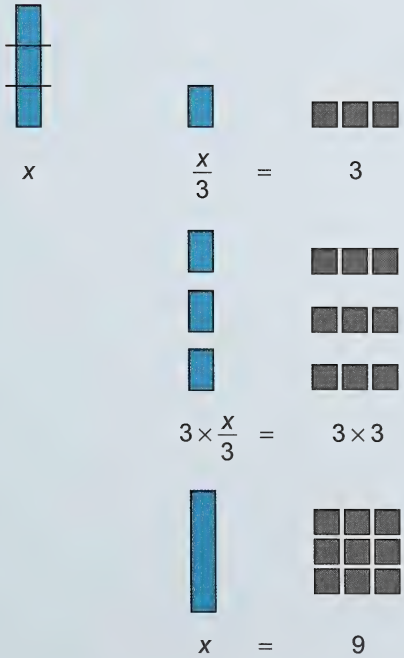
Multiply each side by 9.

$$\frac{x}{9} = 3$$

$$9\left(\frac{x}{9}\right) = 9 \times 3$$

$$x = 27$$

c.



$x \quad \frac{x}{3} = 3$

$3 \times \frac{x}{3} = 3 \times 3$

$x = 9$

Multiply each side by 3.

$$\frac{x}{3} = 3$$

$$3\left(\frac{x}{3}\right) = 3 \times 3$$

$$x = 9$$

d. The numbers are too large to use algebra tiles easily.

Multiply each side by 250.

$$\frac{x}{250} = 4$$

$$250\left(\frac{x}{250}\right) = 250 \times 4$$

$$x = 1000$$

5. Textbook, pages 217 and 218, "Put into Practice," questions 1, 3, 4, 5, and 6

1. Answers will vary. A sample answer is given.

Hey, Joe, did you know you can solve $x+2=3$ in four different ways? I learned this in math today. First, you look at it and see that $1+2=3$. They called this method inspection. Second, you can make it a story. You and I had 3 cookies. I had 2, so you had 1. They called this method representation. Then we modelled the equation with algebra tiles and worked it out. You know, 1 green long plus 2 red squares on one side and 3 red squares on the other. Pop a couple of white squares on each side and pair them up with red squares. What's left? The green long and 1 red square. Finally, they threw algebra at us. You know, using reverse operations. I think it went like this.

$x+2=3$

$x+2-2=3-2$ This is where you subtrct 2 to get rid of the +2.

$x=1$

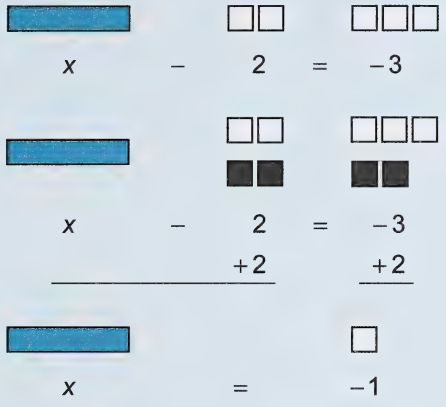
You'll have no trouble with this stuff.

Geoff

3. a. By inspection, $x-4=3$ reveals that $7-4=3$, so $x=7$.

b. A farmer bought a cow. She now has 5 cows. She must have had 4 cows before.

c. Using algebra tiles proves that $x-2=-3$ has the solution $x=-1$.



d. Using the reverse operation gives $x=-3$.

$$\begin{aligned} x+2&=-1 \\ x+2-2&=-1-2 \quad \text{Subtract 2 from each side.} \\ x&=-3 \end{aligned}$$

e. Using the reverse operation gives $x=2$.

$$\begin{aligned} 2x&=4 \\ \frac{2x}{2}&=\frac{4}{2} \quad \text{Divide each side by 2.} \\ x&=2 \end{aligned}$$

f. Using the reverse operation gives $x=-1$.

$$\begin{aligned} 3x&=-3 \\ \frac{3x}{3}&=\frac{-3}{3} \quad \text{Divide each side by 3.} \\ x&=-1 \end{aligned}$$

g. Using the reverse operation gives $x = 8$.

$$\frac{x}{2} = 4$$
$$2\left(\frac{x}{2}\right) = 2 \times 4 \quad \text{Multiply each side by 2.}$$
$$x = 8$$

h. Using the reverse operation gives $x = -6$.

$$\frac{x}{2} = -3$$
$$2\left(\frac{x}{2}\right) = 2 \times (-3) \quad \text{Multiply each side by 2.}$$
$$x = -6$$

4. Methods will vary. The reverse-operation method is shown.

a.	$2x = 16$	$x = 8$	Divide each side by 2.
b.	$y - 7 = 3$	$y = 10$	Add 7 to each side.
c.	$6 + a = 13$	$a = 7$	Subtract 6 from each side.
d.	$\frac{x}{4} = -3$	$x = -12$	Multiply each side by 4.
e.	$8k = -24$	$k = -3$	Divide each side by 8.
f.	$b + 9 = 3$	$b = -6$	Subtract 9 from each side.
g.	$\frac{m}{2} = -5$	$m = -10$	Multiply each side by 2.
h.	$j - 12 = -20$	$j = -8$	Add 12 to each side.

5. Methods will vary. The reverse-operation method is shown.

a.	$4n = 48$	$n = 12$	Divide each side by 4.
b.	$a - 15 = -73$	$a = -58$	Add 15 to each.
c.	$\frac{x}{16} = 5$	$x = 80$	Multiply each side by 16.
d.	$y + 31 = 54$	$y = 23$	Subtract 31 from each side.
e.	$21k = -63$	$k = -3$	Divide each side by 21.

f. $\frac{c}{5} = -18$ $c = -90$ Multiply each side by 5.

g. $p - 24 = 28$ $p = 52$ Add 24 to each side.

h. $x + 16 = -26$ $x = -42$ Subtract 16 from each side.

6. There were 75 candies in the bag.

$$\frac{c}{15} = 5$$

$$15\left(\frac{c}{15}\right) = 15 \times 5 \quad \text{Multiply each side by 15.}$$

$$c = 75$$

Lesson 2: One-Step Equations—Problem Solving

1. Textbook, page 220, "Investigation," questions 1 to 4

1. You are trying to find how many more hours she must work to pay for the snowboard. The variable should represent the number of hours to be worked. Let h represent the number of hours to be worked.

2. $\$6.50h = \$304.95 - \$155.00$ Do the subtraction to find out how much more she needs.

3. $\$6.50h = \$304.95 - \$155.00$

$$\frac{\$6.50h}{\$6.50} = \frac{\$149.95}{\$6.50} \quad \text{Divide each side by \$6.50.}$$

$$h \doteq 23.069\,230\,77$$

4. Ellen will have to work 24 h more. (Working 23 h will leave her \$0.45 short.)

2. Textbook, pages 221 and 222, "Put into Practice," questions 1, 2, 3, 7, 8, and 9

1. $3 \text{ dozen} = 3 \times 12$

$= 36$

Let the amount of meat be m lb.

$$m = \frac{1}{4} \times 36$$

$$m = 9$$

Bob needs 9 lb of meat. The cost will be $9 \times \$2.75 = \$24.75$.

2. The cost of the juice is to be found. Let j be the cost of the juice.

The cost of the lunch was the cost of the sandwich plus the cost of the juice.

$$\$4.85 = \$3.50 + j$$

$$\begin{aligned} \$4.85 &= \$3.50 + j \\ \$4.85 - \$3.50 &= \$3.50 - \$3.50 + j \quad \text{Subtract \$3.50 from each side.} \\ \$1.35 &= j \end{aligned}$$

Alicia paid $1.35 for the juice.

3. How much Jonathan will be paid is to be found. Let p be the amount Jonathan will be paid.

The amount Jonathan will be paid is five-and-a-half times his pay rate.

$$p = \$6.40/\text{h} \times 5.5\,\text{h}$$

$$p = \$6.40/\text{h} \times 5.5\,\text{h}$$
$$p = \$35.20$$

Jonathan will be paid $35.20.

7. The number of cups of apples to be added is what is to be found. Let A be the number of cups of apples to be added.

The number of cups of apples in the pie is 2 plus the number needed.

$$4 = 2 + A$$

$$\begin{aligned} 4 &= 2 + A \\ 4 - 2 &= 2 - 2 + A \quad \text{Subtract 2 from each side.} \\ 2 &= A \end{aligned}$$

Two cups of apples need to be added to the bowl.

8. The price of one loaf of bread is to be found. Let L be the price of one loaf of bread.

The price of three loaves of bread is 3 times the price of one loaf of bread.

$$\$2.79 = 3 \times L$$

$$\$2.79 = 3 \times L$$
$$\frac{\$2.79}{3} = \frac{3 \times L}{3} \quad \text{Divide each side by 3.}$$
$$\$0.93 = L$$

A loaf of bread costs \$0.93.

9. The amount of the tip is to be found. Let T be the tip.

The amount left is the price of the meal plus the tip.

$$\$120.00 = \$102.85 + T$$

$$\$120.00 = \$102.85 + T$$
$$\$120.00 - \$102.85 = \$102.85 - \$102.85 + T \quad \text{Subtract \$102.85 from each side.}$$
$$\$17.15 = T$$

Laura's parents left a \$17.15 tip.

Lesson 3: Solving Two-Step Equations

1. Textbook, pages 227 to 229, "Investigation"

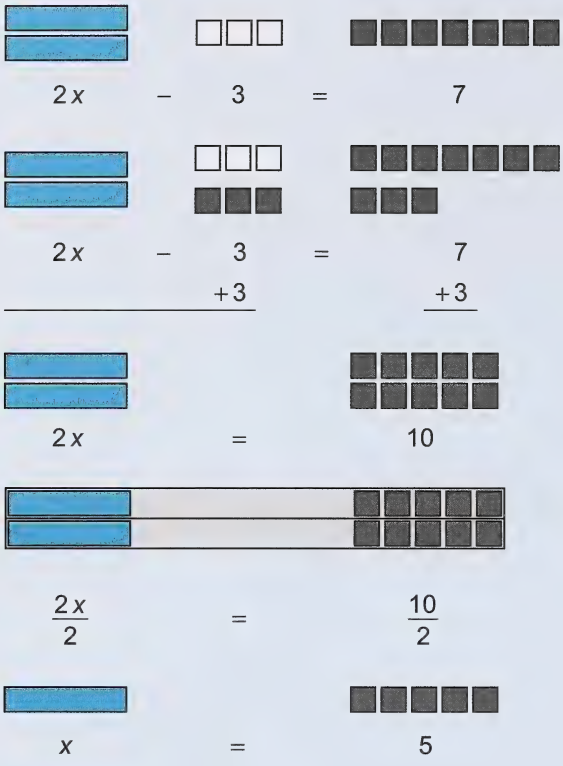
Inspection

Think: $\boxed{10} - 3$ is 7, so $2x = 10$.

$2 \times \boxed{5}$ is 10.

$x = \boxed{5}$

Algebra Tiles



Representation

She gave 3 pieces away.
She must have started with 10 pieces.
She had 5 chocolate bars.

$x = 5$

Reverse Operations

$$2x - 3 = 7$$

$2x - 3 + 3 = 7 + 3$ Add 3 to both sides.

$\frac{2x}{2} = \frac{10}{2}$ Divide both sides by 2.

$$x = 5$$

2. Textbook, pages 229 to 231, "Put into Practice," question 1 and either question 2 or 3

1. Methods will vary. A solution using reverse operations is given for each equation.

a. $\boxed{3x}+5=23$

$\boxed{3x}+5-5=23-5$ Subtract 5 from each side.

$\boxed{3x}=18$

$\frac{3x}{3}=\frac{18}{3}$ Divide each side by 3.

$x=6$

b. $\boxed{6x}-7=-1$

$\boxed{6x}-7+7=-1+7$ Add 7 to each side.

$\boxed{6x}=6$

$\frac{6x}{6}=\frac{6}{6}$ Divide each side by 6.

$x=1$

c. $\boxed{\frac{x}{3}}+5=7$

$\boxed{\frac{x}{3}}+5-5=7-5$ Subtract 5 from each side.

$\boxed{\frac{x}{3}}=2$

$3\left(\frac{x}{3}\right)=3\times 2$ Multiply each side by 3.

$x=6$

d. $\boxed{\frac{x}{2}}-3=6$

$\boxed{\frac{x}{2}}-3+3=6+3$ Add 3 to each side.

$\boxed{\frac{x}{2}}=9$

$2\left(\frac{x}{2}\right)=2\times 9$ Multiply each side by 2.

$x=18$

e. $\boxed{4x} - 9 = 11$

$\boxed{4x} - 9 + 9 = 11 + 9$ Add 9 to each side.

$\frac{4x}{4} = \frac{20}{4}$ Divide each side by 4.

$x = 5$

f. $11 - \boxed{2x} = 5$

$11 - 11 - \boxed{2x} = 5 - 11$ Subtract 11 from each side.

$\frac{-2x}{-2} = \frac{-6}{-2}$ Divide each side by -2.

$x = 3$

g. $\boxed{2x} - 3 = 7$

$\boxed{2x} - 3 + 3 = 7 + 3$ Add 3 to both sides.

$\frac{2x}{2} = \frac{10}{2}$ Divide both sides by 2.

$x = 5$

h. $\boxed{\frac{x}{4}} + 1 = 0$

$\boxed{\frac{x}{4}} + 1 - 1 = 0 - 1$ Subtract 1 from each side.

$\boxed{\frac{x}{4}} = -1$

$4\left(\frac{x}{4}\right) = 4 \times (-1)$ Multiply each side by 4.

$x = -4$

i. $\left[\frac{x}{4}\right]-1=0$

$\left[\frac{x}{4}\right]-1+1=0+1$ Add 1 to each side.

$\left[\frac{x}{4}\right]=1$

$4\left(\frac{x}{4}\right)=4\times1$ Multiply each side by 4.

$x=4$

2. Methods may vary. Solutions using reverse operations follow.

−9	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8	9
R	O	A	S	T	T	U	R	K	E	Y	W	I	T	H	S	A	G	E

a. $\left[\frac{x}{3}\right]-5=-2$

$\left[\frac{x}{3}\right]-5+5=-2+5$ Add 5 to each side.

$\left[\frac{x}{3}\right]=3$

$3\left(\frac{x}{3}\right)=3\times3$ Multiply each side by 3.

$x=9$

b. $[3x]+4=-11$

$[3x]+4-4=-11-4$ Subtract 4 from each side.

$[3x]=-15$

$\frac{3x}{3}=\frac{-15}{3}$ Divide each side by 3.

$x=-5$

c. $\frac{x}{8}+9=9$

$\frac{x}{8}+9-9=9-9$ Subtract 9 from each side.

$\frac{x}{8}=0$

$8\left(\frac{x}{8}\right)=8\times 0$ Multiply each side by 8.

$x=0$

d. $\frac{x}{2}-8=-12$

$\frac{x}{2}-8+8=-12+8$ Add 8 to each side.

$\frac{x}{2}=-4$

$2\left(\frac{x}{2}\right)=2\times(-4)$ Multiply each side by 2.

$x=-8$

e. $6x+17=-1$

$6x+17-17=-1-17$ Subtract 17 from each side.

$6x=-18$

$\frac{6x}{6}=\frac{-18}{6}$ Divide each side by 6.

$x=-3$

f. $9x-2=-20$

$9x-2+2=-20+2$ Add 2 to each side.

$9x=-18$

$\frac{9x}{9}=\frac{-18}{9}$ Divide each side by 9.

$x=-2$

g. $\frac{x}{4} - 10 = -8$

$\frac{x}{4} - 10 + 10 = -8 + 10$ Add 10 to each side.

$\frac{x}{4} = 2$

$4\left(\frac{x}{4}\right) = 4 \times 2$ Multiply each side by 4.

$x = 8$

h. $10x + 6 = 16$

$10x + 6 - 6 = 16 - 6$ Subtract 6 from each side.

$10x = 10$

$\frac{10x}{10} = \frac{10}{10}$ Divide each side by 10.

$x = 1$

i. $4x - 3 = 17$

$4x - 3 + 3 = 17 + 3$ Add 3 to each side.

$4x = 20$

$\frac{4x}{4} = \frac{20}{4}$ Divide each side by 4.

$x = 5$

j. $\frac{x}{7} - 5 = -6$

$\frac{x}{7} - 5 + 5 = -6 + 5$ Add 5 to each side.

$\frac{x}{7} = -1$

$7\left(\frac{x}{7}\right) = 7 \times (-1)$ Multiply each side by 7.

$x = -7$

k. $\frac{x}{2} - 14 = -12$

$\frac{x}{2} - 14 + 14 = -12 + 14$ Add 14 to each side.

$\frac{x}{2} = 2$

$2\left(\frac{x}{2}\right) = 2 \times 2$ Multiply each side by 2.

$x = 4$

l. $\frac{x}{3} + 7 = 4$

$\frac{x}{3} + 7 - 7 = 4 - 7$ Subtract 7 from each side.

$\frac{x}{3} = -3$

$3\left(\frac{x}{3}\right) = 3 \times (-3)$ Multiply each side by 3.

$x = -9$

m. $\frac{x}{3} + 8 = 6$

$\frac{x}{3} + 8 - 8 = 6 - 8$ Subtract 8 from each side.

$\frac{x}{3} = -2$

$3\left(\frac{x}{3}\right) = 3 \times (-2)$ Multiply each side by 3.

$x = -6$

n. $2x - 11 = -5$

$2x - 11 + 11 = -5 + 11$ Add 11 to each side.

$2x = 6$

$\frac{2x}{2} = \frac{6}{2}$ Divide each side by 2.

$x = 3$

o. $\frac{x}{3}+16=18$

$\frac{x}{3}+16-16=18-16$ Subtract 16 from each side.

$\frac{x}{3}=2$

$3\left(\frac{x}{3}\right)=3\times 2$ Multiply each side by 3.

$x=6$

p. $2x+13=5$

$2x+13-13=5-13$ Subtract 13 from each side.

$2x=-8$

$\frac{2x}{2}=\frac{-8}{2}$ Divide each side by 2.

$x=-4$

q. $12x+16=4$

$12x+16-16=4-16$ Subtract 16 from each side.

$12x=-12$

$\frac{12x}{12}=\frac{-12}{12}$ Divide each side by 12.

$x=-1$

r. $\frac{x}{2}-3=-2$

$\frac{x}{2}-3+3=-2+3$ Add 3 to each side.

$\frac{x}{2}=1$

$2\left(\frac{x}{2}\right)=2\times 1$ Multiply each side by 2.

$x=2$

s. $\boxed{2x}+5=-9$

$\boxed{2x}+5-5=-9-5$ Subtract 5 from each side.

$\frac{2x}{2}=\frac{-14}{2}$ Divide each side by 2.

$x=-7$

3. Solution methods may vary. Solutions using the reverse-operation method follow.

15	2	18	23	10	3	12		25	14	16	24			
B	A	N	N	O	C	K		W	I	T	H			
17	20	9	6	11	8	21	13	4		5	19	1	22	7
S	A	S	K	A	T	O	O	N		J	E	L	L	Y

1. $\boxed{4y}-9=7$

$\boxed{4y}-9+9=7+9$ Add 9.

$\boxed{4y}=16$

$\frac{4y}{4}=\frac{16}{4}$ Divide by 4.

$y=4$

2. $\boxed{2a}-4=12$

$\boxed{2a}-4+4=12+4$ Add 4.

$\frac{2a}{2}=\frac{16}{2}$ Divide by 4.

$a=8$

3. $\boxed{\frac{n}{2}}+13=18$

$\boxed{\frac{n}{2}}+13-13=18-13$ Subtract 13.

$\boxed{\frac{n}{2}}=5$

$2\left(\frac{n}{2}\right)=2\times5$ Multiply by 2.

$n=10$

4. $\boxed{3k}-7=26$

$\boxed{3k}-7+7=26+7$ Add 7.

$\boxed{3k}=33$

$\frac{3k}{3}=\frac{33}{3}$ Divide by 3.

$k=11$

5. $\boxed{\frac{d}{4}} + 29 = 32$

$\boxed{\frac{d}{4}} + 29 - 29 = 32 - 29$ Subtract 29.

$\frac{d}{4} = 3$

$4\left(\frac{d}{4}\right) = 4 \times 3$ Multiply by 4.

$d = 12$

6. $\boxed{\frac{x}{5}} - 11 = -8$

$\boxed{\frac{x}{5}} - 11 + 11 = -8 + 11$ Add 11.

$\boxed{\frac{x}{5}} = 3$

$5\left(\frac{x}{5}\right) = 5 \times 3$ Multiply by 5.

$x = 15$

7. $\boxed{8x} - 9 = 7$

$\boxed{8x} - 9 + 9 = 7 + 9$ Add 9.

$\boxed{8x} = 16$ Divide by 8.

$\frac{8x}{8} = \frac{16}{8}$

$x = 2$

8. $\boxed{\frac{b}{2}} + 21 = 28$

$\boxed{\frac{b}{2}} + 21 - 21 = 28 - 21$ Subtract 21.

$\frac{b}{2} = 7$

$2\left(\frac{b}{2}\right) = 2 \times 7$ Multiply by 2.

$b = 14$

9. $\boxed{\frac{h}{2}} + 43 = 55$

$\boxed{\frac{h}{2}} + 43 - 43 = 55 - 43$ Subtract 43.

$\frac{h}{2} = 12$

$2\left(\frac{h}{2}\right) = 2 \times 12$ Multiply by 2.

$h = 24$

10. $\boxed{3a} - 16 = 38$

$\boxed{3a} - 16 + 16 = 38 + 16$ Add 16.

$\boxed{3a} = 54$

$\frac{3a}{3} = \frac{54}{3}$ Divide by 3.

$a = 18$

11. $\boxed{7m}+9=65$

$\boxed{7m}+9-9=65-9$ Subtract 9.

$\boxed{7m}=56$

$\frac{7m}{7}=\frac{56}{7}$ Divide by 7.

$m=8$

12. $\boxed{2y}-7=23$

$\boxed{2y}-7+7=23+7$ Add 7.

$\boxed{2y}=30$

$\frac{2y}{2}=\frac{30}{2}$ Divide by 2.

$y=15$

13. $\boxed{\frac{v}{3}}-38=-32$

$\boxed{\frac{v}{3}}-38+38=-32+38$ Add 38.

$\boxed{\frac{v}{3}}=6$

$3\left(\frac{v}{3}\right)=3\times 6$ Multiply by 6.

$v=18$

14. $\boxed{2t}+7=49$

$\boxed{2t}+7-7=49-7$ Subtract 7.

$\boxed{2t}=42$

$\frac{2t}{2}=\frac{42}{2}$ Divide by 2.

$t=21$

15. $\boxed{5n}-22=63$

$\boxed{5n}-22+22=63+22$ Add 22.

$\boxed{5n}=85$

$\frac{5n}{5}=\frac{85}{5}$ Divide by 5.

$n=17$

16. $\boxed{5w}+18=88$

$\boxed{5w}+18-18=88-18$ Subtract 18.

$\boxed{5w}=70$

$\frac{5w}{5}=\frac{70}{5}$ Divide by 5.

$w=14$

17. $\boxed{\frac{c}{6}}-27=-23$

$\boxed{\frac{c}{6}}-27+27=23+27$ Add 27.

$\boxed{\frac{c}{6}}=4$

$6\left(\frac{c}{6}\right)=6\times 4$ Multiply by 6.

$c=24$

18. $\boxed{4r}+42=86$

$\boxed{4r}+42-42=86-42$ Subtract 42.

$\boxed{4r}=44$

$\frac{4r}{4}=\frac{44}{4}$ Divide by 4.

$r=11$

19. $\boxed{3n} - 38 = 19$

$\boxed{3n} - 38 + 38 = 19 + 38$ Add 7.

$\boxed{3n} = 57$

$\frac{3n}{3} = \frac{57}{3}$ Divide by 3.

$n = 19$

20. $\boxed{\frac{p}{4}} + 64 = 66$

$\boxed{\frac{p}{4}} + 64 - 64 = 66 - 64$ Subtract 64.

$\boxed{\frac{p}{4}} = 2$

$4\left(\frac{p}{4}\right) = 4 \times 2$ Multiply 4.

$p = 8$

21. $\boxed{\frac{y}{9}} - 31 = -29$

$\boxed{\frac{y}{9}} - 31 + 31 = -29 + 31$ Add 31.

$\boxed{\frac{y}{9}} = 2$

$9\left(\frac{y}{9}\right) = 9 \times 2$ Multiply by 9.

$y = 18$

22. $\boxed{10m} - 21 = 19$

$\boxed{10m} - 21 + 21 = 19 + 21$ Add 21.

$\boxed{10m} = 40$

$\frac{10m}{10} = \frac{40}{10}$ Divide by 10.

$m = 4$

23. $\boxed{4c} - 26 = 18$

$\boxed{4c} - 26 + 26 = 18 + 26$ Add 26.

$\boxed{4c} = 44$

$\frac{4c}{4} = \frac{44}{4}$ Divide by 4.

$c = 11$

24. $\boxed{4k} + 15 = 67$

$\boxed{4k} + 15 - 15 = 67 - 15$ Subtract 15.

$\boxed{4k} = 52$

$\frac{4k}{4} = \frac{52}{4}$ Divide by 4.

$k = 13$

25. $\left[\frac{q}{5}\right] + 60 = 64$

$\left[\frac{q}{5}\right] + 60 - 60 = 64 - 60$ Subtract 60.

$\left[\frac{q}{5}\right] = 4$

$5\left(\frac{q}{5}\right) = 4 \times 5$ Multiply by 5.

$q = 20$

Lesson 4: Verifying Answers to Equations

1. Textbook, page 224, "Investigation"

Solution

$\frac{x}{4} = 5$

$4\left(\frac{x}{4}\right) = 4 \times 5$ Multiply each side by 4.

$x = 20$

Verification

	Left Side	Right Side
	$\frac{x}{4}$	5
Substitute 20 for x.	$= \frac{20}{4}$	
	$= 5$	

Since the left side equals the right side, $x = 20$ is the solution for $\frac{x}{4} = 5$.

2. Textbook, page 224, "Put into Practice," questions 1 and 2

1. a. Verification

	Left Side	Right Side
	$x+5$	7
Substitute 2 for x.	$=2+5$	
	$=7$	

Since the left side equals the right side, the solution $x=2$ is correct.

b. Verification

	Left Side	Right Side
	$x-3$	-2
Substitute -1 for x.	$=-1-3$	
	$=-4$	

Since the left side does not equal the right side, the solution $x=-1$ is incorrect. Here is the correct solution.

Solution

$$x-3=-2$$
$$x-3+3=-2+3 \quad \text{Add 3 to each side.}$$
$$x=1$$

Verification

	Left Side	Right Side
	$x-3$	-2
Substitute 1 for x.	$=1-3$	
	$=-2$	

Since the left side equals the right side, the solution is $x=1$.

c. Verification

Left Side	Right Side
$\frac{x}{4}$	2
$=\frac{4}{4}$ Substitute 4 for x.	
$=1$	

Since the left side does not equal the right side, the solution is $x = 4$ is incorrect. Here is the correct solution.

Solution

$$\frac{x}{4} = 2$$

$$4\left(\frac{x}{4}\right) = 4 \times 2 \quad \text{Multiply each side by 4.}$$

$$x = 8$$

Verification

Left Side	Right Side
$\frac{x}{4}$	2
$=\frac{8}{4}$ Substitute 8 for x.	
$=2$	

Since the left side equals the right side, the solution is $x = 8$.

d. Verification

Left Side	Right Side
$x-8$	-2
$=6-8$ Substitute 6 for x.	
$=-2$	

Since the left side equals the right side, the solution $x = 6$ is correct.

e. Verification

	Left Side	Right Side
	$5x$	5
Substitute 0 for x.	$=5\times0$	
	$=0$	

Since the left side does not equal the right side, the solution $x=0$ is incorrect. Here is the correct solution.

Solution

$$5x=5$$

$$\frac{5x}{5}=\frac{5}{5}$$ Divide each side by 5.

$$x=1$$

Verification

	Left Side	Right Side
	$5x$	5
Substitute 1 for x.	$=5\times1$	
	$=5$	

Since the left side equals the right side, the solution is $x=1$.

f. Verification

	Left Side	Right Side
	$2x$	12
Substitute 6 for x.	$=2\times6$	
	$=12$	

Since the left side equals the right side, the solution $x=6$ is correct.

2. a.

$$\boxed{2x} - 8 = 10$$

$$\boxed{2x} - 8 + 8 = 10 + 8 \quad \text{Add 8.}$$

$$\boxed{2x} = 18$$

$$\frac{2x}{2} = \frac{18}{2} \quad \text{Divide by 2.}$$

$$x = 9$$

Left Side	Right Side
$2x - 8$	10
$= (2 \times 9) - 8$	
$= 18 - 8$	
$= 10$	

b.

$$\boxed{\frac{b}{4}} + 5 = 10$$

$$\boxed{\frac{b}{4}} + 5 - 5 = 10 - 5 \quad \text{Subtract 5.}$$

$$\boxed{\frac{b}{4}} = 5$$

$$4\left(\frac{b}{4}\right) = 4 \times 5 \quad \text{Multiply by 4.}$$

$$b = 20$$

Left Side	Right Side
$\frac{b}{4} + 5$	10
$= \frac{20}{4} + 5$	
$= 5 + 5$	
$= 10$	

c.

$$\boxed{3h} + 4 = 10$$

$$\boxed{3h} + 4 - 4 = 10 - 4 \quad \text{Subtract 4.}$$

$$\boxed{3h} = 6$$

$$\frac{3h}{3} = \frac{6}{3} \quad \text{Divide by 3.}$$

$$h = 2$$

Left Side	Right Side
$3h + 4$	10
$= (3 \times 2) + 4$	
$= 6 + 4$	
$= 10$	

d.

$$\boxed{5y} - 5 = 10$$

$$\boxed{5y} - 5 + 5 = 10 + 5 \quad \text{Add 5.}$$

$$\boxed{5y} = 15$$

$$\frac{5y}{5} = \frac{15}{5} \quad \text{Divide by 5.}$$

$$y = 3$$

Left Side	Right Side
$5y - 5$	10
$= (5 \times 3) - 5$	
$= 15 - 5$	
$= 10$	

e. $\boxed{\frac{n}{8}}+15=10$

$\boxed{\frac{n}{8}}+15-15=10-15$ Subtract 35.

$\boxed{\frac{n}{8}}=-5$

$8\left(\frac{n}{8}\right)=8\times(-5)$ Multiply by 8.

$n=-40$

Left Side	Right Side
$\frac{n}{8}+15$	10
$=\frac{-40}{8}+15$	
$=-5+15$	
$=10$	

f. $\boxed{\frac{k}{5}}+13=10$

$\boxed{\frac{k}{5}}+13-13=10-13$ Subtract 22.

$\boxed{\frac{k}{5}}=-3$

$5\left(\frac{k}{5}\right)=5\times(-3)$ Divide by 6.

$k=-15$

Left Side	Right Side
$\frac{k}{5}+13$	10
$=\frac{-15}{5}+13$	
$=-3+13$	
$=10$	

g. $\boxed{6d}+22=10$

$\boxed{6d}+22-22=10-22$ Subtract 22.

$\boxed{6d}=-12$

$\frac{6d}{6}=\frac{-12}{6}$ Divide by 6.

$d=-2$

Left Side	Right Side
$6d+22$	10
$=6\times(-2)+22$	
$=-12+22$	
$=10$	

h. $\boxed{\frac{x}{3}}+1=10$

$\boxed{\frac{x}{3}}+1-1=10-1$ Subtract 1.

$\boxed{\frac{x}{3}}=9$

$3\left(\frac{x}{3}\right)=3\times9$ Multiply by 3.

$x=27$

Left Side	Right Side
$\frac{x}{3}+1$	10
$=\frac{27}{3}+1$	
$=9+1$	
$=10$	

3. Textbook, page 233, "Put into Practice," question 1

1. a. Verification

	Left Side	Right Side
	$2x-6$	-4
Substitute 1 for x.	$=2(1)-6$	
	$=2-6$	
	$=-4$	

Since the left side equals the right side, $x=1$ is the correct solution.

b. Verification

	Left Side	Right Side
	$4x+7$	-21
Substitute -6 for x.	$=4(-6)+7$	
	$=-24+7$	
	$=-17$	

Since the left side does not equal the right side, $x=-6$ is not the correct solution. The correct solution and verification follow.

Solution

$$\boxed{4x}+7=-21$$

$$\boxed{4x}+7-7=-21-7 \quad \text{Subtract 7 from each side.}$$

$$\boxed{4x}=-28$$

$$\frac{4x}{4}=\frac{-28}{4} \quad \text{Divide each side by 4.}$$

$$x=-7$$

Verification

	Left Side	Right Side
	$4x+7$	-21
Substitute -7 for x.	$=4(-7)+7$	
	$=-28+7$	
	$=-21$	

Since the left side equals the right side, $x=-7$ is the correct solution.

c. Verification

	Left Side	Right Side
	$7x+13$	97
Substitute 12 for x.	$=7(12)+13$	
	$=84+13$	
	$=97$	

Since the left side equals the right side, $x=12$ is the correct solution.

d. Verification

	Left Side	Right Side
	$\frac{x}{11}+38$	42
Substitute 44 for x.	$=\frac{44}{11}+38$	
	$=4+38$	
	$=42$	

Since the left side equals the right side, $x=44$ is the correct solution.

e. Verification

Left Side	Right Side
$\frac{x}{6}+10$	1
$=\frac{-66}{6}+10$ Substitute -66 for x.	
$=-11+10$	
$=-1$	

Since the left side does not equal the right side, $x=-66$ is not the correct solution. The correct solution and verification follow.

Solution

$$\boxed{\frac{x}{6}}+10=1$$

$$\boxed{\frac{x}{6}}+10-10=1-10 \quad \text{Subtract 10 from each side.}$$

$$\boxed{\frac{x}{6}}=-9$$

$$6\left(\frac{x}{6}\right)=6\times(-9) \quad \text{Multiply each side by 6.}$$

$$x=-54$$

Verification

Left Side	Right Side
$\frac{x}{6}+10$	1
$=\frac{-54}{6}+10$ Substitute -54 for x.	
$=-9+10$	
$=1$	

Since the left side equals the right side, $x=-54$ is the correct solution.

f. Verification

	Left Side	Right Side
	$12x-50$	46
Substitute 8 for x.	$=(12\times 8)-50$	
	$=96-50$	
	$=46$	

Since the left side equals the right side, $x=8$ is the correct solution.

Lesson 5: Two-Step Equations—Problem Solving

1. Textbook, page 235, "Investigation"

Step 1: Let d be the number of doughnuts left over.

Step 2: Total doughnuts divided by 5 plus 1 is 14.

$$\frac{d}{5}+1=14$$

Step 3: $\boxed{\frac{d}{5}}+1=14$

$\boxed{\frac{d}{5}}+1-1=14-1$ Subtract 1.

$\boxed{\frac{d}{5}}=13$

$5\left(\frac{d}{5}\right)=5\times 13$ Multiply by 5.

$d=65$

Step 4: There were 65 doughnuts left over from the party.

2. Textbook, pages 235 to 238, "Put into Practice," questions 1 to 8 and question 10

1. You need to find the price of a milkshake.

Step 1: Let M be the price of one milkshake.

Step 2: 9 times the price of a milkshake plus 12 times the price of a hamburger is \$42.21.

$$9M + (12 \times \$2.10) = \$42.21$$

Step 3:

$$\boxed{9M} + (12 \times \$2.10) = \$42.21$$

$$\boxed{9M} + \$25.20 = \$42.21$$

$$\boxed{9M} + \$25.20 - \$25.20 = \$42.21 - \$25.20 \quad \text{Subtract \$25.20.}$$

$$\boxed{9M} = \$17.01$$

$$\frac{9M}{9} = \frac{\$17.01}{9} \quad \text{Divide by 9.}$$

$$M = \$1.89$$

Step 4: One milkshake cost \$1.89.

2. You need to find how much each boy paid before adding the tip.

Step 1: Let B be the amount each boy paid before adding the tip.

Step 2: 3 times the amount each boy paid plus the tip is \$45.

$$3B + \$6 = \$45$$

Step 3:

$$\boxed{3B} + \$6 = \$45$$

$$\boxed{3B} + \$6 - \$6 = \$45 - \$6 \quad \text{Subtract \$6.}$$

$$\boxed{3B} = \$39$$

$$\frac{3B}{3} = \frac{\$39}{3} \quad \text{Divide by 3.}$$

$$B = \$13$$

Step 4: Each boy paid \$13 before adding the tip.

3. You need to find how much money Larrissa took into the store.

Step 1: Let L be the amount Larrissa took into the store.

Step 2: The amount left is her starting amount less half the amount less the price of a loaf of bread.

$$\$2.36 = L - \frac{L}{2} - \$1.39$$

Step 3:

$$\$2.36 = L - \frac{L}{2} - \$1.39$$

$$\$2.36 = \boxed{\frac{L}{2}} - \$1.39$$

$$\$2.36 + \$1.39 = \boxed{\frac{L}{2}} - \$1.39 + \$1.39 \quad \text{Add \$1.39.}$$

$$\$3.75 = \boxed{\frac{L}{2}}$$

$$2 \times \$3.75 = 2\left(\frac{L}{2}\right) \quad \text{Multiply by 2.}$$

$$\$7.50 = L$$

Step 4: Larrissa took \$7.50 into the store.

4. You need to find the price of a dozen eggs.

Step 1: Let E be the price of a dozen eggs.

Step 2: The sum of 2 times the price of a dozen eggs plus a litre of milk is \$3.87.

$$\$3.87 = 2E + \$1.49$$

Step 3:

$$\$3.87 = \boxed{2E} + \$1.49$$

$$\$3.87 - \$1.49 = \boxed{2E} + \$1.49 - \$1.49 \quad \text{Subtract \$1.49.}$$

$$\$2.38 = \boxed{2E}$$

$$\frac{\$2.38}{2} = \frac{2E}{2} \quad \text{Divide by 2.}$$

$$\$1.19 = E$$

Step 4: Brian paid \$1.19 for a dozen eggs.

5. You need to find the number of hours Tamara has to work to buy the jeans.

Step 1: Let T be the number of hours Tamara has to work to buy the jeans.

Step 2: The cost of the jeans is her savings plus \$6.65 times the number of hours needed.

$$\$74.90 = \$35 + \$6.65T$$

Step 3:

$$\$74.90 = \$35 + \boxed{\$6.65T}$$

$$\$74.90 - \$35 = \$35 - \$35 + \boxed{\$6.65T} \quad \text{Subtract \$35.}$$

$$\$39.90 = \boxed{\$6.65T}$$

$$\frac{\$39.90}{\$6.65} = \frac{\$6.65T}{\$6.65} \quad \text{Divide by \$6.65.}$$

$$6 = T$$

Step 4: Tamara will have to work 6h more to afford the jeans.

6. You need to find how much Kevin made in tips.

Step 1: Let T be amount of tips Kevin made.

Step 2: One-sixth of his tips plus bus fare is the amount spent.

$$\frac{T}{6} + \$1.75 = \$11.25$$

Step 3:

$$\boxed{\frac{T}{6}} + \$1.75 = \$11.25$$

$$\boxed{\frac{T}{6}} + \$1.75 - \$1.75 = \$11.25 - \$1.75 \quad \text{Subtract \$1.75.}$$

$$\boxed{\frac{T}{6}} = \$9.50$$

$$6\left(\frac{T}{6}\right) = 6 \times \$9.50 \quad \text{Multiply by 6.}$$

$$T = \$57.00$$

Step 4: Kevin made \$57.00 in tips.

7. You need to find how many peanut butter cookies were made.

Step 1: Let P be the number of peanut butter cookies made. (This is also the number of chocolate chip cookies made.)

Step 2: 30 oatmeal raisin cookies plus twice the number of peanut butter cookies is 110.

$$30 + 2P = 110$$

Step 3:

$$30 + \boxed{2P} = 110$$

$$30 - 30 + \boxed{2P} = 110 - 30 \quad \text{Subtract 30.}$$

$$\boxed{2P} = 80$$

$$\frac{2P}{2} = \frac{80}{2} \quad \text{Divide by 2.}$$

$$P = 40$$

Step 4: Johann baked 40 peanut butter cookies.

8. You need to find how many tables were in Tina's section.

Step 1: Let T be the number of tables in Tina's section.

Step 2: The number of birthdays served was one-quarter of her tables less 2.

$$3 = \frac{T}{4} - 2$$

Step 3:

$$3 = \boxed{\frac{T}{4}} - 2$$

$$3 + 2 = \boxed{\frac{T}{4}} - 2 + 2 \quad \text{Add 2.}$$

$$5 = \boxed{\frac{T}{4}}$$

$$4 \times 5 = 4\left(\frac{T}{4}\right) \quad \text{Multiply by 4.}$$

$$20 = T$$

Step 4: There were 20 tables in Tina's section.

10. You need to find how many cans of pop were ordered.

Step 1: Let C be the number of cans of pop ordered.

Step 2: The cost of pop was \$0.63 times the number ordered, less \$0.63 times the number returned.

$$\$272.16 = \$0.63C - (\$0.63 \times 18)$$

Step 3:

$$\$272.16 = \boxed{\$0.63C} - (\$0.63 \times 18)$$

$$\$272.16 = \boxed{\$0.63C} - \$11.34$$

$$\$272.16 + \$11.34 = \boxed{\$0.63C} - \$11.34 + \$11.34 \quad \text{Add \$11.34.}$$

$$\$283.50 = \boxed{\$0.63C}$$

$$\frac{\$283.50}{\$0.63} = \frac{\$0.63C}{\$0.63} \quad \text{Divide by \$0.63.}$$

$$450 = C$$

Step 4: The school ordered 450 cans of pop.

Lesson 6: Reading and Interpreting Graphs

1. Textbook, pages 96 to 98, "Investigation," questions 2 to 5

2. a. Andrew spends about half of his money on entertainment.

b. Half of \$50 is \$25. Andrew spent \$25 on entertainment.

$$\$50 \div 2 = \$25$$

c. "Food and Snacks" and "Clothes" seem to be the same size.

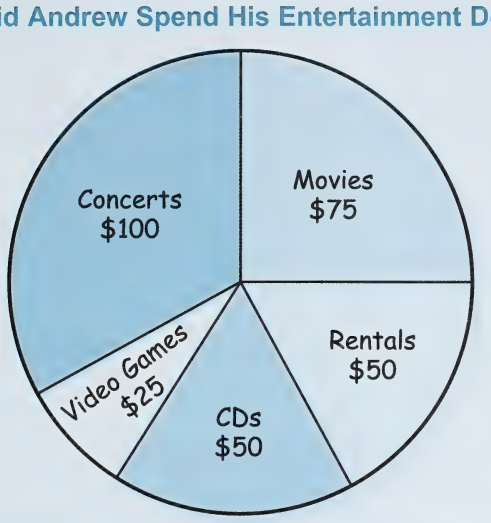
d. Andrew spent about an eighth of his money on school supplies. This is about \$6.

$$\$50 \div 8 \doteq \$6$$

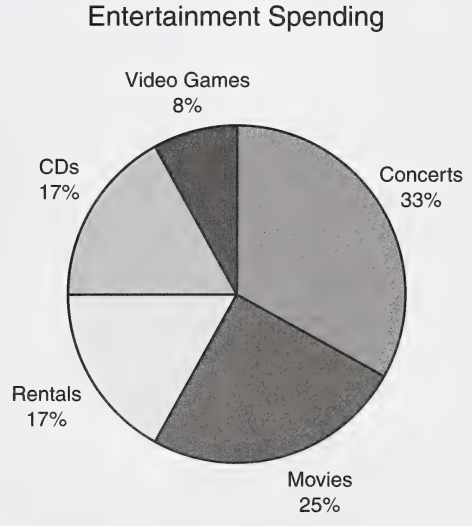
e. Andrew spent about \$19 on clothes and food and snacks.

$$\$50 - \$25 - \$6 = \$19$$

3. a. You can draw a pie chart by hand or use a spreadsheet. Samples of both follow.



	A	B
1	Concerts	$100
2	Movies	$75
3	Rentals	$50
4	CDs	$50
5	Video Games	$25
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		



b. The total amount spent was $300.

$$\$100 + \$75 + \$50 + \$50 + \$25 = \$300$$

4. a. There were 60 votes recorded.

$$7+9+6+11+14+5+8=60$$

b. 54-40 received the most votes.

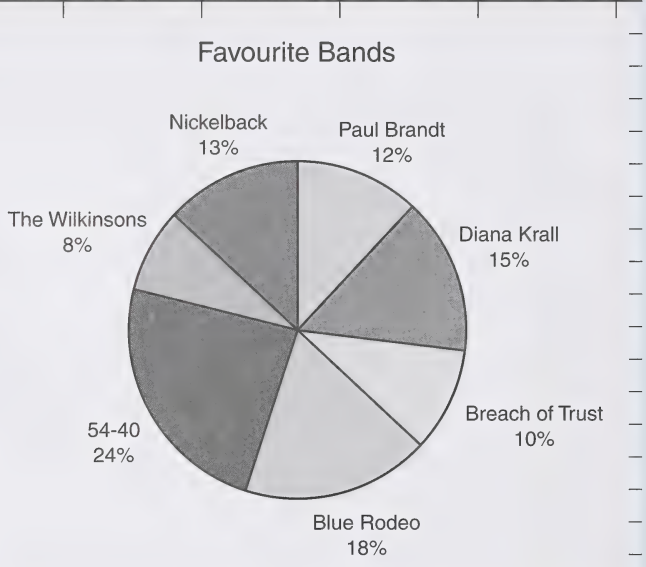
c. The Wilkinsons received the fewest votes.

d. There were 30 people voting.

$$60\div 2=30$$

5. Enter the data from the favourite band survey into a spreadsheet program. Use the chart feature to make a pie graph.

	A	B	C	D	E	F	G
1	Paul Brandt	7					
2	Diana Krall	9					
3	Breach of Trust	6					
4	Blue Rodeo	11					
5	54-40	14					
6	The Wilkinsons	5					
7	Nickelback	8					
8							
9							
10							
11							
12							
13							
14							
15							
16							
17							
18							
19							



2. a. **Textbook, page 99, "Put into Practice," questions 1.b., 1.d., and 1.e.**

1. b.

People Visiting the Library on Monday Morning	
Age Range in Years	**Number of People**
0–15	2
16–30	5
31–45	4
46–60	8
61–75	6

d.

Boxes of Recycled Paper Collected	
School Grade	**Number of Boxes**
3	4
4	5
5	2
6	8

e.

Number of Breaths in 15-s Interval During a Walk to the Library	
Time Walking in Minutes	**Number of Breaths in 15 s**
1	4
2	8
3	6
4	8
5	10
6	8
7	8
8	8
9	9
10	10
11	9
12	8

b. Textbook, pages 101 to 112, "Put into Practice," questions 3, 4, 5, 8, 12, 13, 14.a. to e., and 15

3. a. Histograms involve one variable. You put the data into one of several groups. Then you count how many are in each group. The height of the rectangle you draw is proportional to the number of items in the group. Bar graphs have several measurements of different items.

b. The groups of distances are along the bottom. The number in each group is along the side.

c. The 0–500 km distance group is most frequent.

d. Answers will vary. A sample answer follows.

It could be used to plan the cost of providing free travel to the awards.

e. Answers will vary. A sample answer follows.

It doesn't tell how the people travelled. It doesn't tell when the people travelled. It doesn't tell what award people are getting.

4. a. The arts category has the most awards.

b. Science and sports seem to be about the same. They are the next largest category.

5. a. There were 36 people surveyed.

$$9+3+2+4+3+6+4+5=36$$

b. Most people wanted to win an arts award.

c. Business and health are equally popular.

d. Science and education are equally popular.

e. Six people wanted an award in law.

8. a. A complete note represents two playings of the type of music it is beside.

b. Answers will vary. A sample answer follows.

The icons are eye-catching. The information is clearly labelled and laid out. Comparing the categories is easy.

12. a. Sets and studio are about the same. Director, story, and rentals are also about the same.

b. Answers will vary. A sample answer follows.

The pieces of the pie for the items mentioned are about the same size.

c. The actors get about a quarter of the budget. In a $10 million movie, the actors would get about $2.5 million.

d. Answers will vary. A sample answer follows.

The actors that are the most popular will be paid the most.

e. Answers will vary. A sample answer follows.

This is not fair, but it is good business.

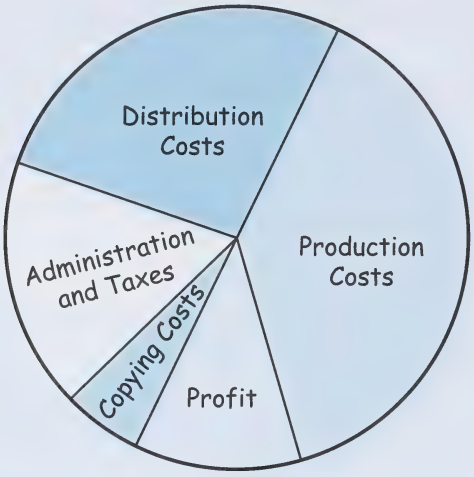
f. Answers will vary. A sample answer follows.

It is surprising that the director gets such a big piece of the pie.

g. Answers will vary. A sample answer follows.

The information is about portions. Circle graphs are a good way to display this.

13. a. Answers will vary. A sample answer follows.



b. Answers will vary. Pretty much any way of labelling the graph is fine.

14. a. The number of movies attended is also shown.

b. The 12–17 age group attends the most movies.

c. The 50–64 and 65+ groups attend the fewest movies.

d. Answers will vary. A sample answer follows.

Young people have fewer commitments, so they can spend more time on entertainment.

e. Answers will vary.

15. **a.** Japan and Switzerland have the highest ticket prices.

b. Singapore and South Africa have the lowest ticket prices.

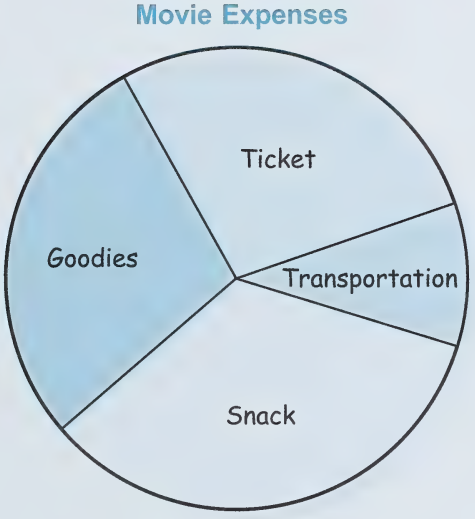
c. Answers will vary. A sample answer follows.

It is not too surprising. Wealthy countries would have higher prices.

d. Answers will vary. A sample answer follows.

There would be costs for transportation. There would be "goodies" at the movie. There would likely be a snack after the movie.

e.



Review

1. Textbook, pages 65 to 67, questions 5.a., 5.b., 6, 8, and 11

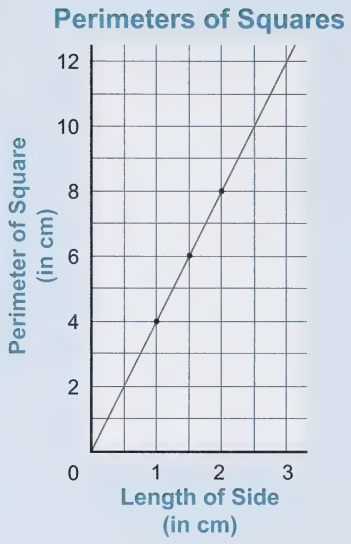
5. a.

Picture	Number of Circles	Number of Triangles
1	3	1
2	5	2
3	7	3
4	9	4

b. The number of circles increases by 2 for each added triangle. There will be a multiply by 2 in the mathematical sentence. $2 \times 1 = 2$ is one too small. You add 1 to get the correct result. The equation is $C = 2T + 1$.

6. a.

Length of Side (cm)	Perimeter (cm)
1	4
1.5	6
2	8



b. For every move to the right 1 unit, the graph goes up 4 units. Or the perimeter is 4 times the length of a side.

c. $P = 4s$

d. Answers will vary. A sample answer follows.

You could measure more squares of varying sizes and see that they fit the rule.

8. a. $2n - 1 = 7$ matches C. (Twice a number subtract 1 gives 7.)
b. $x \div 3 = 5$ matches D. (A number divided by 3 equals 5.)
c. $2y = 11$ matches B. (A number multiplied by 2 equals 11.)
d. $t + 6 = 14$ matches A. (A number increased by 6 equals 14.)

11. $P = 2\ell + 2w$

$P = 2(8 \text{ cm}) + 2(6 \text{ cm})$ Substitute 8 cm for ℓ and 6 cm for w.
$= 16 \text{ cm} + 12 \text{ cm}$
$= 28 \text{ cm}$

The perimeter would be 28 cm.

2. a. The completed magic square follows. The explanation of the values is also given.

23	2	17
8	14	20
11	26	5

$14 \times 3 = 42$, so the magic number for this square is 42.

$42 - 17 - 14 = 11$
$42 - 17 - 5 = 20$
$42 - 20 - 14 = 8$
$42 - 11 - 8 = 23$
$42 - 14 - 26 = 2$

b. The magic sum for the given magic square would be 90.

3. Pick a number. Let the number be x.

Double your number and add 3.

$$2x + 3$$

Double the result and add 10.

$$2(2x + 3) + 10 = 4x + 6 + 10$$
$$= 4x + 16$$

Divide the result by 4.

$$\frac{4x + 16}{4} = x + 4$$

Subtract the original number.

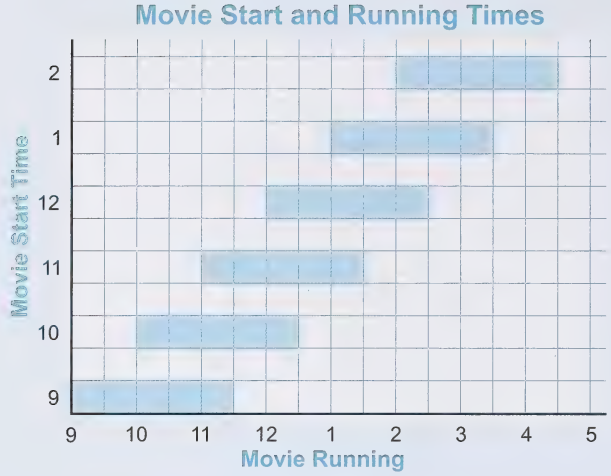
$$x - x + 4 = 4$$

4. Textbook, pages 115 and 116, questions 10 and 11

10. a. The most popular type of show is the sit com.
b. The least popular is the survival type of show.

11. a. The most popular start times are 9:00, 12:00, and 1:00.

b. There would be 18 movies still on at 2:15 A.M.



At 2:15 A.M., there are
2 movies from 2:00 A.M.,
8 movies from 1:00 A.M., and
8 movies from 12:00 A.M.
still showing.

5. a. Solutions for the equations follow.

i. $w - 6 = 27$

$w - 6 + 6 = 27 + 6$ Add 6 to each side.

$w = 33$

ii. $c + 3 = -11$

$c + 3 - 3 = -11 - 3$ Subtract 3 from each side.

$c = -14$

iii. $\frac{x}{17} = 3$

$17\left(\frac{x}{17}\right) = 17 \times 3$ Multiply each side by 17.

$x = 51$

iv. $4y = 64$

$\frac{4y}{4} = \frac{64}{4}$ Divide each side by 4.

$y = 16$

b. Solutions for the equations follow.

i. $\boxed{3q} - 3 = 9$

$\boxed{3q} - 3 + 3 = 9 + 3$ Add 3 to each side.

$\boxed{3q} = 12$

$\frac{3q}{3} = \frac{12}{3}$ Divide each side by 3.

$q = 4$

ii. $\boxed{8r} + 11 = 59$

$\boxed{8r} + 11 - 11 = 59 - 11$ Subtract 11 from each side.

$\boxed{8r} = 48$

$\frac{8r}{8} = \frac{48}{8}$ Divide each side by 8.

$r = 6$

iii. $\boxed{\frac{m}{5}} + 7 = 9$

$\boxed{\frac{m}{5}} + 7 - 7 = 9 - 7$ Subtract 7 from each side.

$\boxed{\frac{m}{5}} = 2$

$5\left(\frac{m}{5}\right) = 5 \times 2$ Multiply each side by 5.

$m = 10$

iv. $\boxed{\frac{k}{12}} - 6 = -1$

$\boxed{\frac{k}{12}} - 6 + 6 = -1 + 6$ Add 6 to each side.

$\boxed{\frac{k}{12}} = 5$

$12\left(\frac{k}{12}\right) = 12 \times 5$ Multiply each side by 12.

$k = 60$

c. i. Verification

Left Side	Right Side
$4q$	68
$=4\times17$	
$=68$	

Substitute 17 for q.

Since the left side equals the right side, $q=17$ is the solution to $4q=68$.

ii.

Left Side	Right Side
$8s-5$	27
$=(8\times3)-5$	
$=24-5$	
$=19$	

Substitute 3 for s.

Since the left side does not equal the right side, $s=3$ is not the solution to $8s-5=27$.

Solution

$$\boxed{8s}-5=27$$
$$\boxed{8s}-5+5=27+5 \quad \text{Add 5 to each side.}$$
$$\boxed{8s}=32$$
$$\frac{8s}{8}=\frac{32}{8} \quad \text{Divide each side by 8.}$$
$$s=4$$

Verification

Left Side	Right Side
$8s-5$	27
$=(8\times4)-5$	
$=32-5$	
$=27$	

Substitute 4 for s.

Since the left side equals the right side, the solution is $s=4$.

6. Textbook, pages 286 and 287, questions 7 and 10

7. Shannon would know that the individual prices add to \$6.90, so the savings would be \$0.95. She should tell him he would save 95 cents.

$$\begin{aligned} \text{individual costs} - \text{combination costs} &= \text{savings} \\ (\$3.95 + \$2.95) - \$5.95 &= \text{savings} \\ \$6.90 - \$5.95 &= \text{savings} \\ \$0.95 &= \text{savings} \end{aligned}$$

10. a. Jason should pay $\$19.95 + 12.95 = \$32.90$ for 3 buckets of wings.

b. Jason should pay $\$19.95 + 19.95 = \$39.90$ for 4 buckets of wings. This is $\$39.90 - \$32.90 = \$7.00$ more than for 3 buckets. The fourth bucket of wings costs him \$7.00.

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